Numerical Modeling of Mixing Processes - What Can LES Offer?

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Objective

- To evaluate whether LES can be used effectively to model fluid behavior in engineering applications
- Outline
  - Brief Summary of Turbulence Models
  - Introduction to Large Eddy Simulation (LES)
  - Examples
    - Bluff Body Jet
    - HEV Static Mixer

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Modeling Turbulence

- Turbulence is a 3D transient phenomenon
  - Fluctuations cover a wide range of time and length scales
- Turbulence models range from approximate to highly rigorous:
  - steady-state isotropic models
  - transient 3D models of entire spectrum
- Models are incorporated into the Navier-Stokes equations using a variety of methods

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The Turbulence Spectrum

- Many scales of turbulent eddies exist:
  - Large eddies contain most of the turbulent kinetic energy
    - Scale sizes are on the order of the flow passages
  - Energy cascades from large to small eddies
  - Small eddies dissipate the energy they receive from larger eddies in the spectrum
- Difficulty in turbulence modeling is trying to accurately capture the contributions of all scales in the spectrum

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Direct Numerical Simulation (DNS)

- Navier-Stokes equations are solved on a fine grid using a small time-step
- Goal is to capture the smallest turbulence scales
  - Large scales are captured as well
- Result is accurate, 3D, transient behavior
- Great for simple flows, but computationally intensive
  - Not suited to industrial applications with cpu resources available today

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The Cost of DNS

- The number of grid points per dimension needed to resolve the small scales is

$$N_{1D} \sim \mathrm{Re}_t^{3/4}, \quad \mathrm{Re}_t = \frac{r\sqrt{k}\,\ell}{m}$$

- The number of grid points needed for a 3D DNS simulation is

$$N_{3D} \sim \mathrm{Re}_t^{9/4}$$

- The overall cost, including time step, of the computational effort[1] is $\sim Re_t^3$

[1]Reynolds, W.C. Turbulence at the Crossroads, pp. 313-342 (1990)

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RANS Turbulence Models 1

- Velocities are described by an equilibrium (v_o) and fluctuating (*v'*) contribution:

  $v_i = v_{oi} + v_i'$

- Momentum equations are rewritten, then time-averaged **(Reynolds Averaged Navier-Stokes equations)**
  - Averaging eliminates terms with v' as a factor
  - Terms with $v_i'v_j'$ remain
  - These *Reynolds stresses* are computed with a turbulence model
  - Impact on transport equations is through the effective viscosity: $m_{eff} \sim m_t + m_o$ (1 and 2 equation models)

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RANS Turbulence Models 2

- Many flavors exist, such as:
  - *k-e*: Robust, popular 2-equation model using constants taken from simple, high Re flows
    - isotropic turbulence effects

      m_{eff} is a scalar
  - RSM: 5-equation (2D) or 7-equation (3D) model
    - non-isotropic turbulence effects makes this suitable for highly swirling flows

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Large Eddy Simulation (LES)

- LES is midway between DNS and RANS in terms of
  - rigor
  - computational requirement
- Spectrum of turbulent eddies in the Navier-Stokes equations is “filtered”
  - The filter is a function of the grid size
  - small eddies are removed, and modeled using a *subgrid-scale (SGS)* model
  - large eddies are retained, and solved for directly using a transient calculation

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Filtered Variables

- A variable, ϕ(x’), is filtered using a filter function, G

$$\tilde{f}(\mathbf{x}) = \int_D f(\mathbf{x}')G(\mathbf{x},\mathbf{x}')d\mathbf{x}'$$

- G is a function of the cell volume

$$G(\mathbf{x},\mathbf{x}') = \begin{cases} 1/V & \text{for } \mathbf{x}' \in n \\ 0 & \text{otherwise} \end{cases}$$

Thus

$$\tilde{f}(\mathbf{x}) = \frac{1}{V}\int_n f(\mathbf{x}')d\mathbf{x}', \qquad \mathbf{x}' \in V$$

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Filtered Transport Equations

- The filtered continuity and momentum equations use filtered variables:

$$\frac{\partial r}{\partial t} + \frac{\partial r\tilde{u}_j}{\partial x_j} = 0$$

and

$$\frac{\partial r\tilde{u}_i}{\partial t} + \frac{\partial r\tilde{u}_i\tilde{u}_j}{\partial x_j} = -\frac{\partial \tilde{p}}{\partial x_i} + \frac{\partial t_{ij}}{\partial x_j} + \frac{\partial s_{ij}}{\partial x_j}$$

t_{ij} is the filtered stress tensor

s_{ij} are the subgrid-scale Reynolds stresses

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Subgrid-Scale (SGS) Modeling

- SGS Reynolds stresses are modeled by

$$s^{s}_{ij} - \frac{1}{3} dij\, s^{s}_{kk} = -2m_t S_{ij}$$

where m_t is the subgrid-scale eddy viscosity and S_{ij} is the rate of strain tensor

- Two models in FLUENT 5 are:

Smagorinsky SGS model

$$m_t = r L^2 \sqrt{2S_{ij}S_{ij}}$$

$$L = \min\left(k\, d, C_s V^{\frac{1}{3}}\right)$$

RNG SGS model

$$m_t = m\left[1 + H\left(\frac{m_s^2 m_{tot}}{m^3} - C\right)\right]^{1/3}$$

$$m_s = \left(0.157V^{\frac{1}{3}}\right)^2 \sqrt{2S_{ij}S_{ij}}$$

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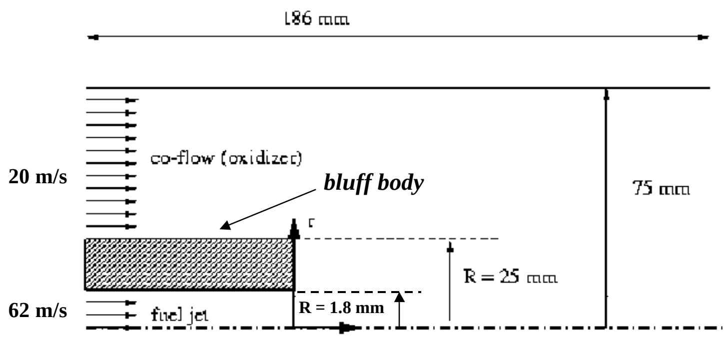
Bluff-Body Jet

- Cold flow in a bluff-body coaxial burner
- A range of length and time scales exists



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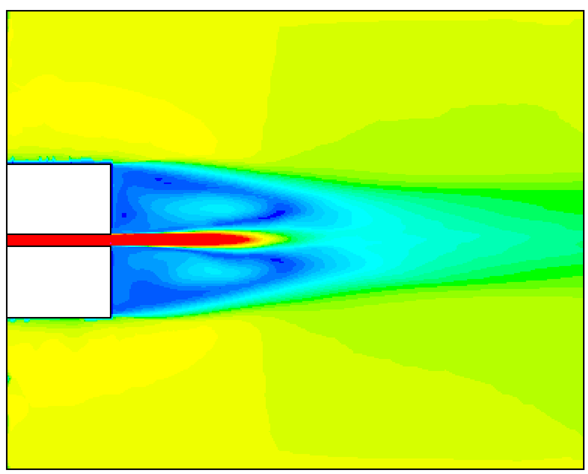
Bluff Body Jet Flow Pattern

Time averaged flow pattern from LES simulations.



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Bluff-Body Jet Axial Velocities

- Transient, 3D solution done with LES*
- Steady-state, 2D axisymmetric solutions done with *k-e* and RSM**
- Comparison with experiment

	V_{axial}(x/D=0.8)	V_{axial}(x/D=1.3)
Experiment	33.5 m/s	12.7 m/s
LES *(t-averaged)*	35.2	13.2
RSM	26.4	7.5
k-e	25.5	7.7

*420, 000 cells ** 30,000 cells

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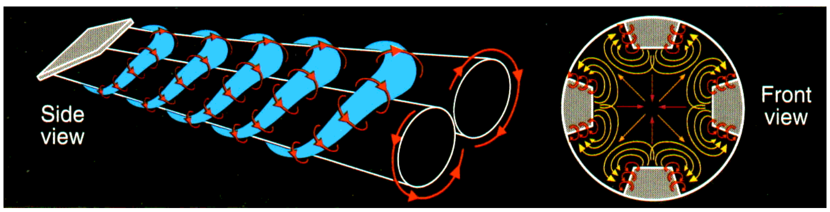
HEV Static Mixer

- Circular or square cross-section pipe with sets of tabs mounted on the walls
- Flow around tabs is unsteady, with counter-rotating longitudinal vortices, and hairpin vortices



Source: "Kenics Static Mixers" brochure, 1996.

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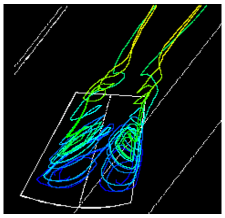
Previous Models

- Assumptions:
  - Eight-fold symmetry.
  - Steady state flow with RANS model
- Results:
  - Longitudinal vortices observed
- Disadvantages:
  - Hairpin vortices not observed
  - Under-prediction of mixing near center
  - No material exchange between areas surrounding tabs



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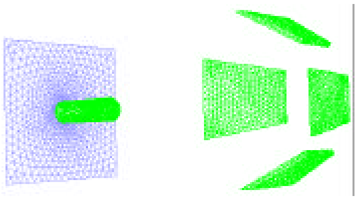
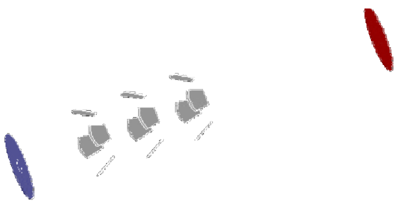
Geometries Studied

- Two models studied
  - Square duct
    - 0.1x0.1x1 m^3
    - Air at 30 m/s
    - Re ~ 200k
  - Cylindrical pipe
    - D = 0.05 m
    - Water at 0.12 m/s
    - Re ~ 5000
- Both models:
  - 500k cells
  - Unstructured grid





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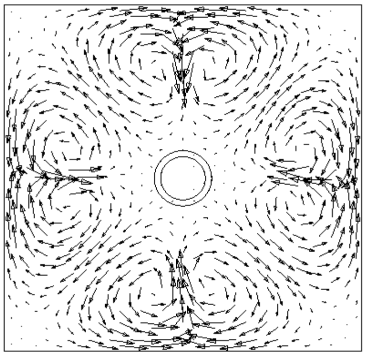
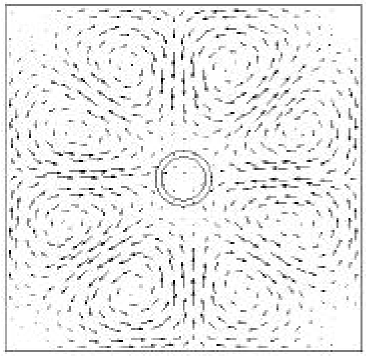
Square Duct Results: RNG k-e

Longitudinal vortices are symmetric and stable.



Between 3rd and 4th tabs



2-D downstream of last tabs

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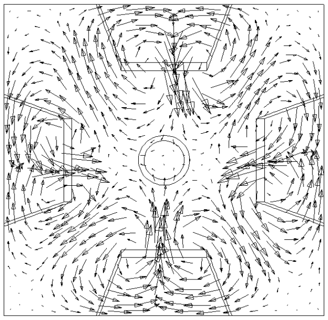
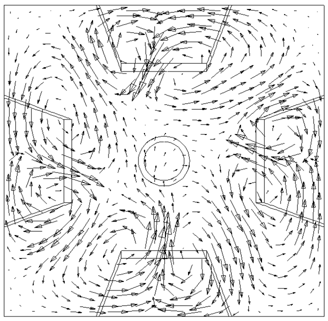
Square Duct Results: LES
Longitudinal Vortices - 1

Between third and fourth sets of tabs



T = 0.1429 s



T = 0.1486 s

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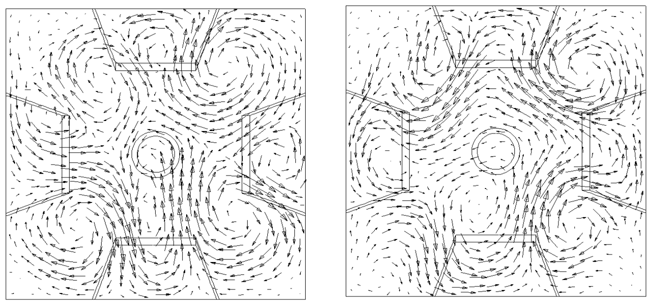
Square Duct Results: LES
Longitudinal Vortices - 2

2-D downstream of last set of tabs



T = 0.1429 s   T = 0.1486 s

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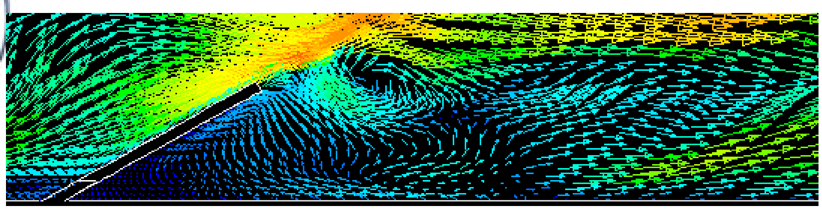
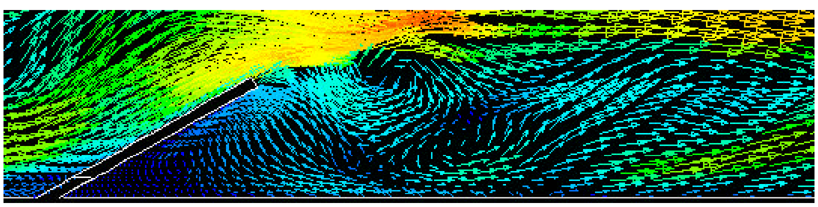
Cylindrical Pipe Results: LES
Hairpin Vortices - 1



T = 6.40 s



T = 6.43 s

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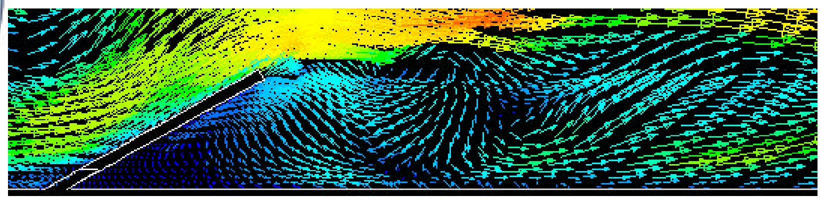
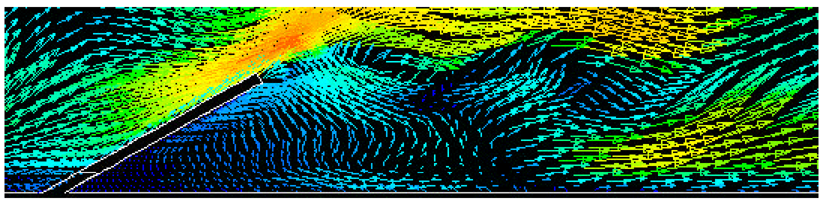
Cylindrical Pipe Results: LES

Hairpin Vortices - 2



T = 6.46 s



T = 6.53 s

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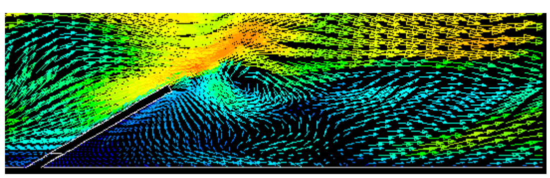
Hairpin Vortex Animation



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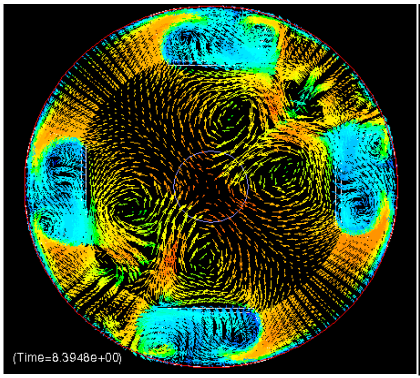
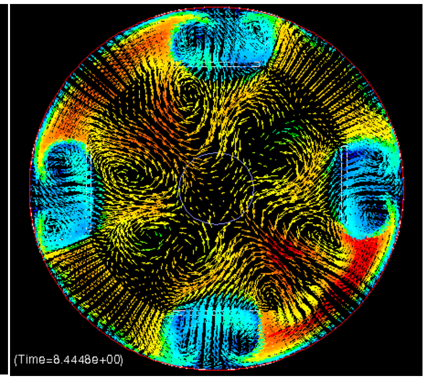
Cylindrical Pipe Results: LES
Longitudinal Vortices - 1

Cylindrical - At Tip of Last Set of Tabs





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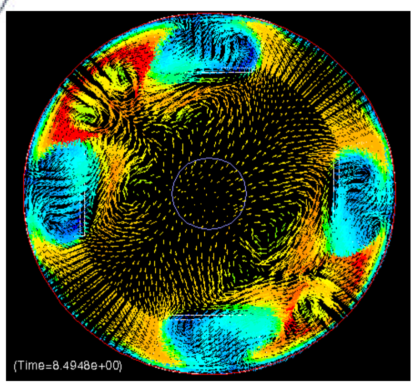
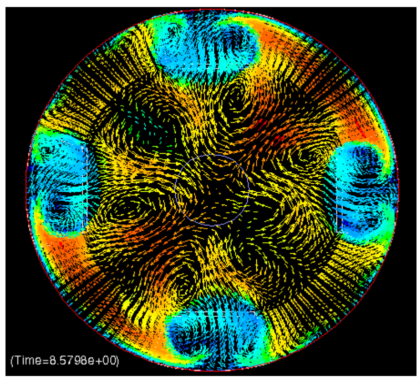
Cylindrical Pipe Results: LES
Longitudinal Vortices - 2

Cylindrical - At Tip of Last Set of Tabs





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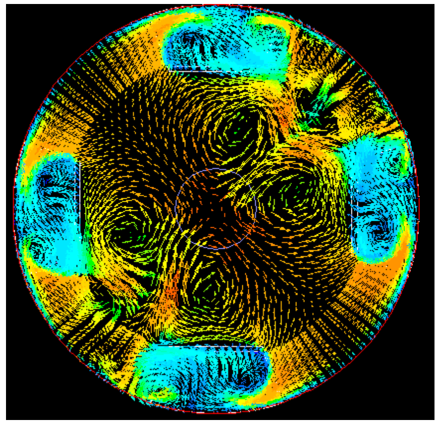
Longitudinal Vortices Animation



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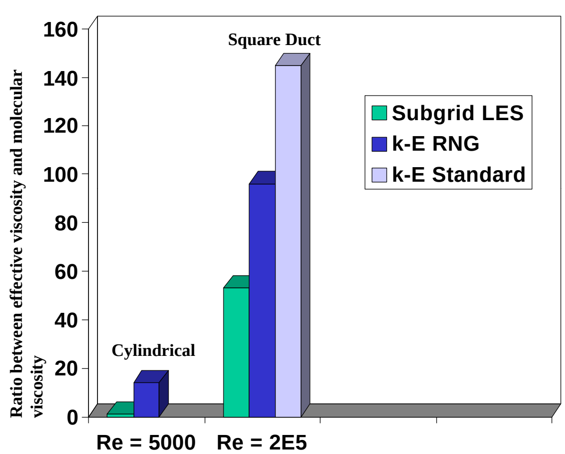
Effective Viscosity Comparison



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HEV Results Discussion

- LES predicts unsteady vortex system including transient hairpin vortices, as also seen in experiments
- Interaction between vortices causes material exchange between tabs, and between the center and tabs
- Practical issues:
  - Calculation time for 1500 time steps on the order of a week on 350 MHz P2 PC
  - 500k node model requires 0.5 GB of RAM
  - Data files ~ 50MB each (compressed)

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Summary

- LES is a transient turbulence model that falls midway between RANS and DNS models
- Time-averaged results better predicted the axial flow pattern in a bluff-body jet example
- Transient results showed hairpin and longitudinal vortices in an HEV mixer, the former long observed experimentally, but not predicted with CFD until now
- LES has potential benefit for engineering applications, and is within reach computationally

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