Lecture 17 - Eulerian-Granular Model

Applied Computational Fluid Dynamics

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Contents

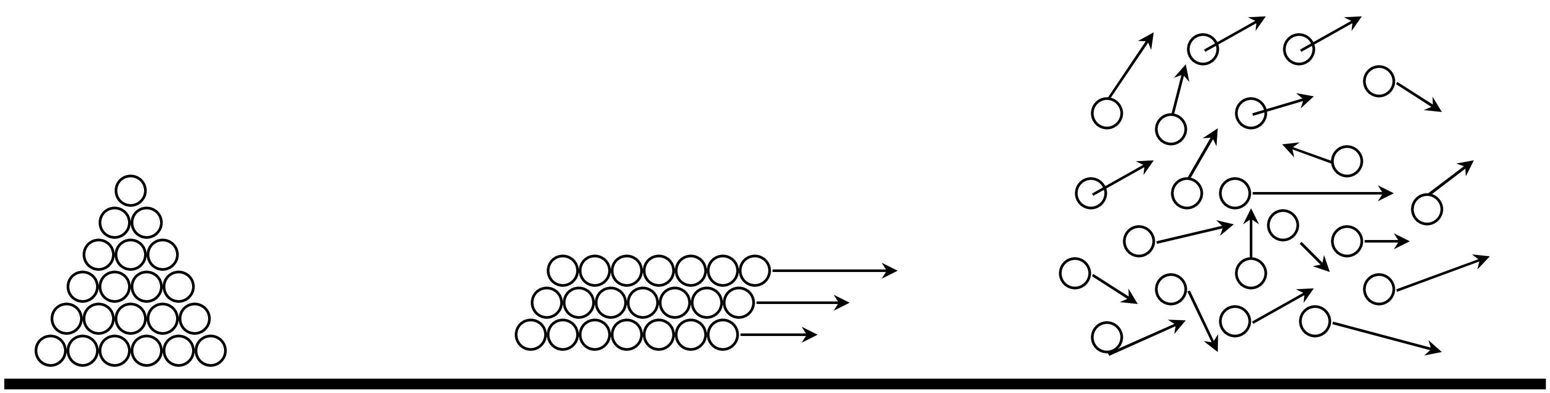
- Overview.
- Description of granular flow.
- Momentum equation and constitutive laws.
- Interphase exchange models.
- Granular temperature equation.
- Solution algorithms for multiphase flows.
- Examples.

Overview

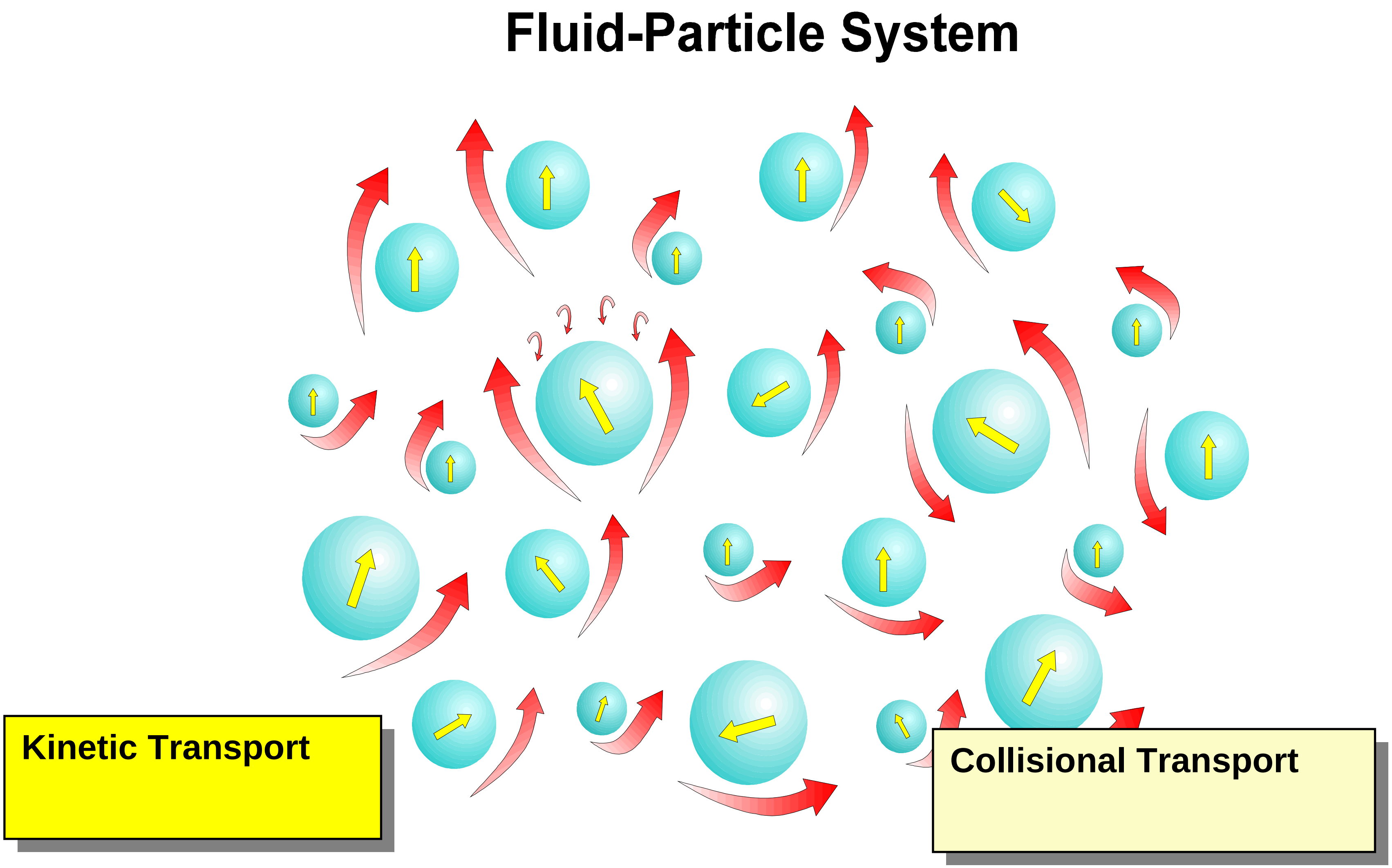
- The fluid phase must be assigned as the primary phase.
- Multiple solid phases can be used to represent size distribution.
- Can calculate granular temperature (solids fluctuating energy) for each solid phase.
- Calculates a solids pressure field for each solid phase.
  - All phases share fluid pressure field.
  - Solids pressure controls the solids packing limit.

Granular flow regimes

Elastic Regime	Plastic Regime	Viscous Regime.
Stagnant	Slow flow	Rapid flow
Stress is strain dependent	Strain rate independent	Strain rate dependent
Elasticity	Soil mechanics	Kinetic theory



Kinetic theory of granular flow



Granular multiphase model: description

- Application of the kinetic theory of granular flow
  Jenkins and Savage (1983), Lun et al. (1984), Ding and Gidaspow (1990).
- Collisional particle interaction follows Chapman-Enskog approach for dense gases (Chapman and Cowling, 1970).
  - Velocity fluctuation of solids is much smaller than their mean velocity.
  - Dissipation of fluctuating energy due to inelastic deformation.
  - Dissipation also due to friction of particles with the fluid.

Granular multiphase model: description (2)

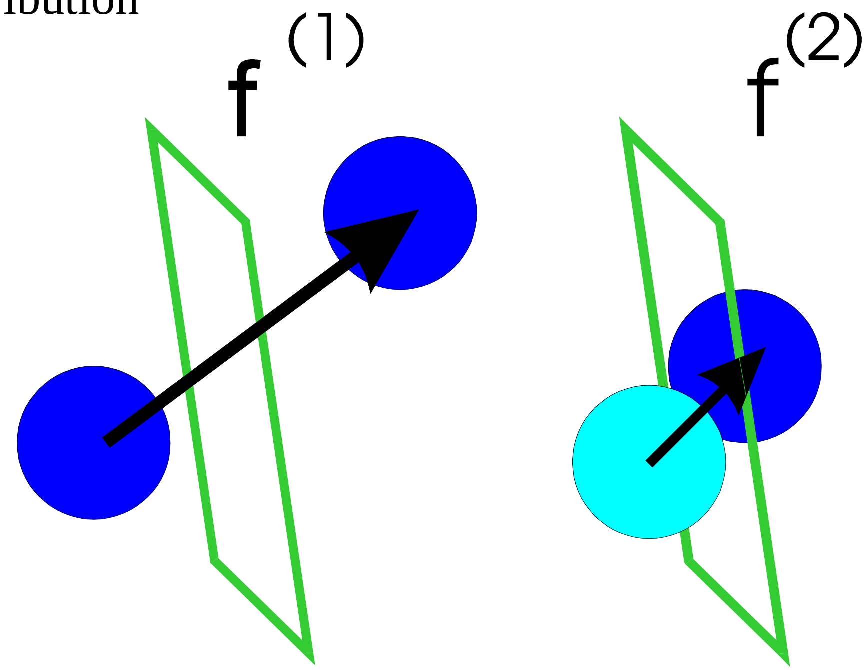
- Particle velocity is decomposed into a mean $\vec{C}$ local velocity and a superimposed fluctuating random velocity $\vec{u}_s$.
- A "granular" temperature is associated with the random fluctuation velocity:

$$\frac{3}{2}\theta = \frac{1}{2}\overline{\vec{C}\cdot\vec{C}}$$

Gas molecules and particle differences

- Solid particles are a few orders of magnitude larger.
- Velocity fluctuations of solids are much smaller than their mean velocity.
- The kinetic part of solids fluctuation is anisotropic.
- Velocity fluctuations of solids dissipates into heat rather fast as a result of inter particle collision.
- Granular temperature is a byproduct of flow.

Analogy to kinetic theory of gases



Granular multiphase model: description

- Several transport mechanisms for a quantity Ψ within the particle phase:
  - Kinetic transport during free flight between collision
    Requires velocity distribution function f^{1}.
  - Collisional transport during collisions
    Requires pair distribution function f^{2}.

- Pair distribution function is approximated by taking into account the radial distribution function $g_o(\vec{r},\sigma)$ into the relation between and f^1 *and* f^2.

Continuity and momentum equations

- Applying Enskog's kinetic theory for dense gases gives for:
  - Continuity equation for the granular phase.

$$\frac{\partial}{\partial t}(\alpha_s \rho_s) + \nabla \cdot (\alpha_s \rho_s \vec{u}_s) = \dot{m}_{fs}$$

Mass transfer

  - Granular phase momentum equation.

$$\frac{\partial}{\partial t}(\alpha_s \rho_s \vec{u}_s) + \nabla \cdot (\alpha_s \rho_s \vec{u}_s \vec{u}_s) = -\alpha_s \nabla p_f + \nabla \cdot \overline{\tau}_s + \sum_{s=1}^{n} (\vec{R}_{fs} + \dot{m}_{fs} \vec{u}_{fs}) + \vec{F}_s$$

Fluid pressure

Solid stress tensor

Phase interaction term

Constitutive equations

- Constitutive equations needed to account for interphase and intraphase interaction:
  - Solids stress $\boxed{\nabla \cdot \bar{\tau}_s}$ Accounts for interaction within solid phase. Derived from granular kinetic theory

$$\bar{\tau}_s = -P_s \bar{I} + 2\alpha_s \mu_s \bar{S} + \alpha_s (\lambda_s - \tfrac{2}{3}\mu_s)\nabla \cdot \vec{u}_s \bar{I}$$

where,

$\bar{S} = \tfrac{1}{2}\left(\nabla \vec{u}_s + (\nabla \vec{u}_s)^T\right)$ Strain rate

P_s Solids Pressure

g_o Radial distribution function

λ_s, μ_s Solids bulk and shear viscosity

Constitutive equations: solids pressure

- Pressure exerted on the containing wall due to the presence of particles.
- Measure of the momentum transfer due to streaming motion of the particles:

$$P_s = \alpha_s \rho_s \theta_s (\omega + 2(1+e_s)\alpha_s g_{os})$$

  - Gidaspow and Syamlal models: $\omega = 1$
  - Sinclair model: $\omega = (1 + \frac{d_s}{6\alpha_s D\sqrt{2}})$

Constitutive equations: radial function

- The radial distribution function $g_0(\alpha_s)$ is a correction factor that modifies the probability of collision close to packing.
- Expressions for $g_0(\alpha_s)$:

Ding and Gidaspow, Sinclair.

$$g_o(\alpha_s) = \left(1 - \left(\frac{\alpha_s}{\alpha_{s,\max}}\right)^{\frac{1}{3}}\right)^{-1}, \alpha_{s,\max} = 0.65$$

Syamlal et al.

$$g_o(\alpha_s) = \frac{1}{1-\alpha_s} + \frac{3\alpha_s}{2(1-\alpha_s)^2}$$

Constitutive equations: solids viscosity

- The solids viscosity:
  - Shear viscosity arises due translational (kinetic) motion and collisional interaction of particles:

$$\mu_s = \mu_{s,coll} + \mu_{s,kin} \qquad \eta = (1+\alpha_s)/2$$

  - Collisional part:
    - Gidaspow and Syamlal models:

$$\mu_{s,coll} = \frac{8}{5}\alpha_s \rho_s d_s g_{os} \eta \left(\frac{\theta_s}{\pi}\right)^{1/2}$$

    - Sinclair model:

$$\mu_{s,col} = \frac{5 d_s \rho_s (\theta_s \pi)^{1/2}}{96\alpha_s}\left(\left[\frac{8\alpha_s}{5(2-\eta)}\right]\left[1+\frac{8}{5}\eta(3\eta-2)\alpha_s g_{os}\right]+\frac{768}{25\pi}\eta\alpha_s^2 g_{os}\right)$$

Constitutive equations: solids viscosity

- Kinetic part:
  - Syamlal model: $$\mu_{s,kin} = \frac{\alpha_s d_s \rho_s (\theta_s \pi)^{1/2}}{12(2-\eta)} \left[1 + \frac{8}{5}\eta(3\eta - 2)\alpha_s g_{os} \right]$$
  - Gidaspow model: $$\mu_{s,kin} = \frac{5 d_s \rho_s (\theta_s \pi)^{1/2}}{96 \eta g_{os}} \left[1 + \frac{8}{5} g_{os} \alpha_s \eta \right]$$
  - Sinclair model: $$\mu_{s,kin} = \frac{5 d_s \rho_s (\theta_s \pi)^{1/2}}{96 \alpha_s \eta (2-\eta) g_{os}} \omega \left[1 + \frac{8}{5}\eta(3\eta - 2)\alpha_s g_{os} \right]$$

Constitutive equations: bulk viscosity

- Bulk viscosity accounts for resistance of solid body to dilatation:

$$\lambda_s = \frac{4}{3}\alpha_s \rho_s d_s g_{os} (1+e_s)\left(\frac{\theta_s}{\pi}\right)^{1/2}$$

  - α_s volume fraction of solid.
  - e_s coefficient of restitution.
  - d_s particle diameter.

Plastic regime: frictional viscosity

- In the limit of maximum packing the granular flow regime becomes incompressible. The solid pressure decouples from the volume fraction.
- In frictional flow, the particles are in enduring contact and momentum transfer is through friction. The stresses are determined from soil mechanics (Schaeffer, 1987).
- The frictional viscosity is:

$$\mu_{s,frict} = \frac{P_s \sin\varphi}{2\sqrt{I_2}}$$

- The effective viscosity in the granular phase is determined from the maximum of the frictional and shear viscosities:

$$\mu_s = \max\left\lfloor \mu_{s,coll} + \mu_{s,kin}, \mu_{s,frict} \right\rfloor$$

Momentum equation: interphase forces

- Interaction between phases. $\sum_{s=1}^{n}(\vec{R}_{fs} + \dot{m}_{fs}\vec{u}_{fs}) = 0$

- Formulation is based on forces on a single particle corrected for effects such as concentration, clustering particle shape and mass transfer effects. The sum of all forces vanishes.
  - Drag: caused by relative motion between phases; K_{fs} is the drag between fluid and solid; K_{ls} is the drag between particles

$$\sum_{l=1}^{n}(K_{ls}(\vec{u}_l - \vec{u}_s)) + K_{fs}(\vec{u}_f - \vec{u}_s) = 0$$

General form for the drag term:

$$K_{fs} = \alpha_s \rho_s \frac{f_{drag}}{\tau_{fs}}$$

With particle relaxation time:

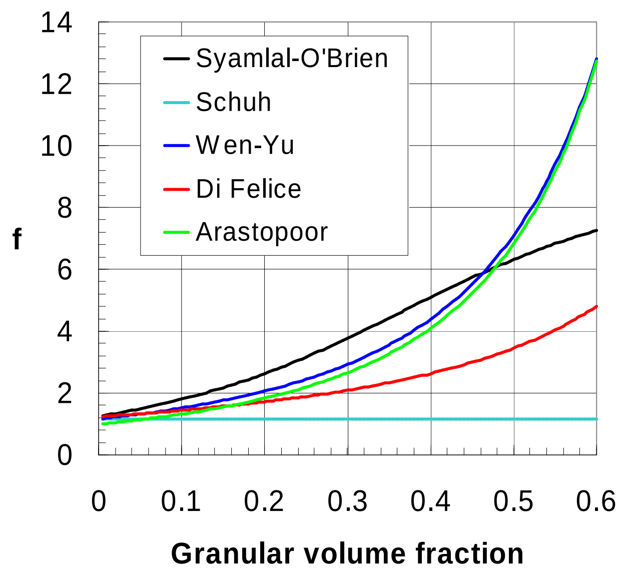
$$\tau_{fs} = \frac{\rho_s d_s^{\ 2}}{18\mu_f}$$

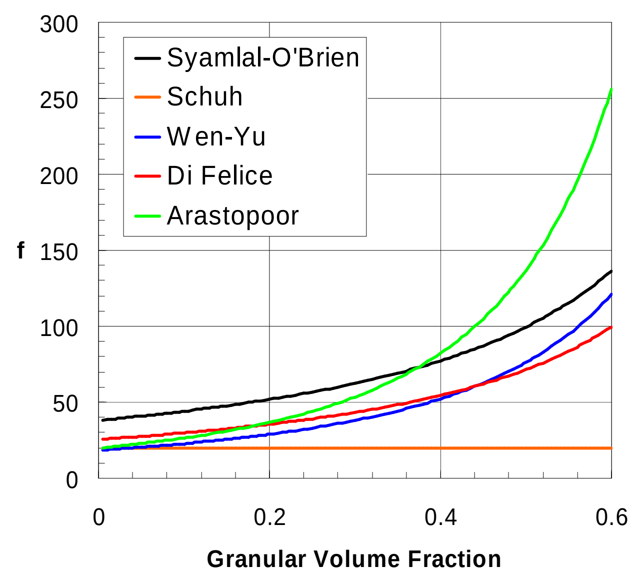
Momentum: interphase exchange models

- Fluid-solid momentum interaction, expressions for f_{drag}.
  - Arastopour et al (1990).
  - Di Felice (1994).
  - Syamlal and O'Brien (1989).
  - Wen and Yu (1966).
- Drag based on Richardson and Zaki (1954) and/or Ergun (1952).
  - use the one that correctly predicts the terminal velocity in dilute flow.
  - in bubbling beds ensure that the minimum fluidized velocity is correct.
  - It depends strongly on the particle diameter: correct diameter for non-spherical particles and/or to include clustering effects.

Comparison of drag laws

- A comparison of the fluid-solid momentum interaction, f_{drag}, for:
  - Relative Reynolds number of 1 and 1000.
  - Particle diameter 0.001 mm.





Particle-particle drag law

- Solid-solid momentum interaction.
  - Drag function derived from kinetic theory (Syamlal et al, 1993).

$$K_{lm} = \frac{3(1+e_{lm})(\frac{\pi}{2} + C_{lm}\frac{\pi^2}{8})\alpha_l \rho_l \alpha_m \rho_m (d_l + d_m)^2 g_{olm}}{2\pi(\rho_l d_l^3 + \rho_m d_m^3)} |\vec{u}_l - \vec{u}_m|$$

$$g_{olm} = \frac{1}{\alpha_f} + \frac{3 d_m d_l}{\alpha_f^2 (d_l + d_m)} \sum_{k=1}^{M} \frac{\alpha_k}{d_k}$$

Momentum: interphase exchange models

- Virtual mass effect: caused by relative acceleration between phases Drew and Lahey (1990).

$$K_{vm,fs} = C_{vm}\alpha_s\rho_f\left((\frac{\partial \vec{u}_f}{\partial t} + \vec{u}_f \cdot \nabla \vec{u}_f) - (\frac{\partial \vec{u}_s}{\partial t} + \vec{u}_s \cdot \nabla \vec{u}_s) \right)$$

- Lift force: caused by the shearing effect of the fluid onto the particle Drew and Lahey (1990).

$$K_{k,fs} = C_L\alpha_s\rho_f(\vec{u}_f - \vec{u}_s)\times(\nabla\times\vec{u}_f)$$

- Other interphase forces are: Basset Force, Magnus Force, Thermophoretic Force, Density Gradient Force.

Granular multiphase model: mass transfer

- Unidirectional mass transfer: $\dot{m}_{fs}$
- Defines positive mass flow is specified constant rate of rate per unit volume from phase f to phase s,
  - $\dot{m}_{fs}$ proportional to: $\dot{r}\alpha_f\rho_s$
  - particle shrinking or swelling.
    - e.g., rate of burning of particle.
  - Heat transfer modeling can be included via UDS.

Granular temperature equations

- Granular temperature. $\theta = \frac{1}{3}\vec{C}\cdot\vec{C}$

$$\frac{3}{2}\left\{\frac{\partial}{\partial t}(\alpha_s\rho_s\theta_s)+\nabla\cdot(\alpha_s\rho_s\vec{u}_s\theta_s)\right\} = \overbrace{\bar{\bar{\tau}}_s:\nabla\vec{u}_s}^{\text{Production term}} + \ldots$$

$$+\overbrace{\nabla\cdot(\kappa_{\theta s}\nabla\theta_s)}^{\text{Diffusion term}}-\underbrace{\gamma_s}_{\text{Dissipation term due to inelastic collisions}}+\underbrace{\phi_{lm}+\phi_{fs}}_{\text{Exchange terms}}$$

Constitutive equations: granular temperature

- Granular temperature for the solid phase is proportional to the kinetic energy of the random motion of the particles.
  - $\boxed{\bar{\tau}_s : \nabla \vec{u}_s}$ represents the generation of energy by the solids stress tensor.
  - $\boxed{\nabla \cdot (\kappa_{\theta s} \nabla \theta_s)}$ represents the diffusion of energy.
  - $\kappa_{\theta s}$ Granular temperature conductivity.

Constitutive equations: granular temperature

- Granular temperature conductivity.
  - Syamlal:

$$\kappa_{\theta_s} = \frac{15\alpha_s \rho_s d_s \sqrt{\theta_s \pi}}{4(41-33\eta)}[1+\frac{12}{5}\alpha_s g_{os}\eta^2(4\eta-3) + \frac{16}{15\pi}(41-33\eta)\eta\alpha_s g_{os}]$$

  - Gidaspow:

$$\kappa_{\theta_s} = \frac{75\rho_s d_s \sqrt{\theta_s \pi}}{384\eta g_{os}}[1+\frac{12}{5}\alpha_s g_{os}\eta]^2 \quad + 2\alpha_s^2 \rho_s d_s (1+e_s) g_{os} \sqrt{\frac{\theta_s}{\pi}}$$

  - Sinclair:

$$\kappa_{\theta_s} = (\kappa_{\theta_s})_{syamlal} + \frac{25\rho_s d_s \sqrt{\theta_s \pi}}{16\eta g_{os}} \omega \left[\frac{1+\frac{12}{5}\eta^2(4\eta-3)\alpha_s g_{os}}{41-33\eta} \right]$$

Constitutive equations: granular temperature

- γ_{θ_s} represents the dissipation of energy due to inelastic collisions.
  - Gidaspow: $\gamma_{\theta_s} = 3(1-e_s^2)\alpha_s^2 \rho_s g_{os} \theta_s \left[\frac{4}{\pi}\sqrt{\frac{\theta}{\pi}} - \nabla \vec{u}_s \right]$
  - Syamlal and Sinclair: $\gamma_{\theta_s} = \frac{12(1-e_s)g_{os}}{d_s\sqrt{\pi}} \rho_s \alpha_s \theta_s^{3/2}$ Lun et al (1984)
- Here ϕ_{lm} represents the energy exchange among solid phases (UDS).

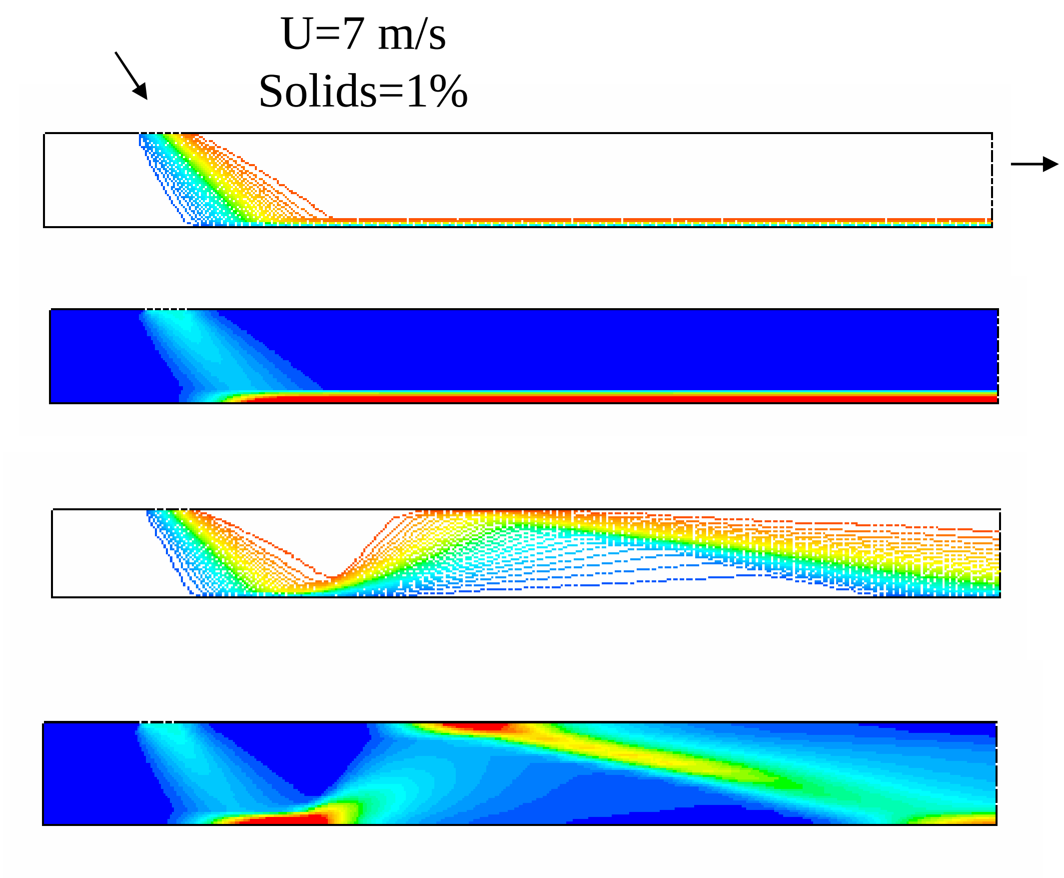
Constitutive equations: granular temperature

- ϕ_{fs} represents the energy exchange between the fluid and the solid phase.
  - Laminar flows: $\phi_{fs} = -3K_{fs}\theta_s$
  - Dispersed turbulent flows:
    - Sinclair: $$\phi_{fs} = K_{fs}(\sqrt{2k_f}\sqrt{3\theta} - 2k_f)$$
    - Other models:

$$\phi_{fs} = K_{fs}(2k_f - < u'_{pi}, u'_{fi} >)$$

Test case for Eulerian granular model

- Contours of solid stream function and solid volume fraction when solving with Eulerian-Eulerian model.
- Contours of solid stream function and solid volume fraction when solving with Eulerian-Granular model.



Solution guidelines

- All multiphase calculations:
  - Start with a single-phase calculation to establish broad flow patterns.
- Eulerian multiphase calculations:
  - Copy primary phase velocities to secondary phases.
  - Patch secondary volume fraction(s) as an initial condition.
  - For a single outflow, use OUTLET rather than PRESSURE-INLET; for multiple outflow boundaries, must use PRESSURE-INLET.
  - For circulating fluidized beds, avoid symmetry planes (they promote unphysical cluster formation).
  - Set the “false time step for underrelaxation” to 0.001.
  - Set normalizing density equal to physical density.
  - Compute a transient solution.

Summary

- The Eulerian-granular multiphase model has been described in the section.
- Separate flow fields for each phase are solved and the interaction between the phases modeled through drag and other terms.
- The Eulerian-granular multiphase model is applicable to all particle relaxation time scales and Includes heat and mass exchange between phases.
- Several kinetic theory formulations available:
    - Gidaspow: good for dense fluidized bed applications.
    - Syamlal: good for a wide range of applications.
    - Sinclair: good for dilute and dense pneumatic transport lines and risers.