Lecture 17 - Eulerian-Granular Model

Applied Computational Fluid Dynamics

Instructor: André Bakker
Contents

• Overview.
• Description of granular flow.
• Momentum equation and constitutive laws.
• Interphase exchange models.
• Granular temperature equation.
• Solution algorithms for multiphase flows.
• Examples.
Overview

• The fluid phase must be assigned as the primary phase.
• Multiple solid phases can be used to represent size distribution.
• Can calculate granular temperature (solids fluctuating energy) for each solid phase.
• Calculates a solids pressure field for each solid phase.
  – All phases share fluid pressure field.
  – Solids pressure controls the solids packing limit.
Granular flow regimes

<table>
<thead>
<tr>
<th>Elastic Regime</th>
<th>Plastic Regime</th>
<th>Viscous Regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stagnant</td>
<td>Slow flow</td>
<td>Rapid flow</td>
</tr>
<tr>
<td>Stress is strain dependent</td>
<td>Strain rate independent</td>
<td>Strain rate dependent</td>
</tr>
<tr>
<td>Elasticity</td>
<td>Soil mechanics</td>
<td>Kinetic theory</td>
</tr>
</tbody>
</table>
Kinetic theory of granular flow

Fluid-Particle System

Kinetic Transport

Collisional Transport
Granular multiphase model: description

- Application of the kinetic theory of granular flow
  - Velocity fluctuation of solids is much smaller than their mean velocity.
  - Dissipation of fluctuating energy due to inelastic deformation.
  - Dissipation also due to friction of particles with the fluid.
Granular multiphase model: description (2)

- Particle velocity is decomposed into a mean $\bar{C}$ local velocity and a superimposed fluctuating random velocity $\bar{u}_s$.
- A “granular” temperature is associated with the random fluctuation velocity:

$$\frac{3}{2} \theta = \frac{1}{2} \bar{C} \cdot \bar{C}$$
Gas molecules and particle differences

- Solid particles are a few orders of magnitude larger.
- Velocity fluctuations of solids are much smaller than their mean velocity.
- The kinetic part of solids fluctuation is anisotropic.
- Velocity fluctuations of solids dissipates into heat rather fast as a result of inter particle collision.
- Granular temperature is a byproduct of flow.
Analogy to kinetic theory of gases

- Velocity distribution function
- Pair distribution function

Free streaming

Collision

Collisions are brief and momentarily. No interstitial fluid effect.
Granular multiphase model: description

- Several transport mechanisms for a quantity $Ψ$ within the particle phase:
  - Kinetic transport during free flight between collision
    Requires velocity distribution function $f^1$.
  - Collisional transport during collisions
    Requires pair distribution function $f^2$.

- Pair distribution function is approximated by taking into account the radial distribution function $g_o(\mathbf{r}, \sigma)$ into the relation between and $f^1$ and $f^2$. 
Continuity and momentum equations

- Applying Enskog’s kinetic theory for dense gases gives for:
  - Continuity equation for the granular phase.
    \[
    \frac{\partial}{\partial t} (\alpha_s \rho_s \vec{u}_s) + \nabla \cdot (\alpha_s \rho_s \vec{u}_s \vec{u}_s) = \dot{m}_{fs}
    \]
  - Granular phase momentum equation.
    \[
    \frac{\partial}{\partial t} (\alpha_s \rho_s \vec{u}_s \vec{u}_s) + \nabla \cdot (\alpha_s \rho_s \vec{u}_s \vec{u}_s \vec{u}_s) = -\alpha_s \nabla p_f + \nabla \cdot \vec{\tau}_s + \sum_{s=1}^{n} (\vec{R}_{fs} + \dot{m}_{fs} \vec{u}_s \vec{u}_s) + \vec{F}_s
    \]
Constitutive equations

- Constitutive equations needed to account for interphase and intraphase interaction:
  - Solids stress \( \nabla \cdot \tau_s \) Accounts for interaction within solid phase. Derived from granular kinetic theory

\[
\tau_s = -P_s I + 2\alpha_s \mu_s \bar{S} + \alpha_s (\lambda_s - \frac{2}{3} \mu_s) \nabla \cdot \bar{u}_s I
\]

where,
\[
\bar{S} = \frac{1}{2} \left( \nabla \bar{u}_s + (\nabla \bar{u}_s)^T \right)
\]
Strain rate
\[
P_s
\]
Solids Pressure
\[
g_o
\]
Radial distribution function
\[
\lambda_s, \mu_s
\]
Solids bulk and shear viscosity
Constitutive equations: solids pressure

- Pressure exerted on the containing wall due to the presence of particles.
- Measure of the momentum transfer due to streaming motion of the particles:

\[ P_s = \alpha_s \rho_s \theta_s (\omega + 2(1 + e_s)\alpha_s g_{os}) \]

  - Gidaspow and Syamlal models: \( \omega = 1 \)
  
  - Sinclair model: \( \omega = (1 + \frac{d_s}{6\alpha_s D\sqrt{2}}) \)
The radial distribution function $g_0(\alpha_s)$ is a correction factor that modifies the probability of collision close to packing.

Expressions for $g_0(\alpha_s)$:

**Ding and Gidaspow, Sinclair.**

$$g_0(\alpha_s) = \left(1 - \left(\frac{\alpha_s}{\alpha_{s,\text{max}}}\right)^{\frac{1}{3}}\right)^{-1}, \alpha_{s,\text{max}} = 0.65$$

**Syamlal et al.**

$$g_0(\alpha_s) = \frac{1}{1-\alpha_s} + \frac{3\alpha_s}{2(1-\alpha_s)^2}$$
Constitutive equations: solids viscosity

- The solids viscosity:
  - Shear viscosity arises due translational (kinetic) motion and collisional interaction of particles:
    \[ \mu_s = \mu_{s,coll} + \mu_{s,kin} \quad \eta = (1 + \alpha_s) / 2 \]
  - Collisional part:
    - Gidaspow and Syamlal models:
      \[ \mu_{s,coll} = \frac{8}{5} \alpha_s \rho_s d_s g \eta \left( \frac{\theta_s}{\pi} \right)^{1/2} \]
    - Sinclair model:
      \[ \mu_{s,col} = \frac{5d_s \rho_s (\theta_s \pi)^{1/2}}{96\alpha_s} \left[ \frac{8\alpha_s}{5(2 - \eta)} \left[ 1 + \frac{8}{5} \eta(3\eta - 2) \alpha_s g \right] + \frac{768}{25\pi} \eta \alpha_s^2 g \right] \]
Constitutive equations: solids viscosity

- Kinetic part:
  - Syamlal model:
    \[
    \mu_{s,\text{kin}} = \frac{\alpha_s d_s \rho_s (\theta_s \pi)^{1/2}}{12(2-\eta)} \left[ 1 + \frac{8}{5} \eta(3\eta - 2) \alpha_s g_{os} \right]
    \]
  - Gidaspow model:
    \[
    \mu_{s,\text{kin}} = \frac{5d_s \rho_s (\theta_s \pi)^{1/2}}{96\eta g_{os}} \left[ 1 + \frac{8}{5} g_{os} \alpha_s \eta \right]
    \]
  - Sinclair model:
    \[
    \mu_{s,\text{kin}} = \frac{5d_s \rho_s (\theta_s \pi)^{1/2}}{96\alpha_s \eta(2-\eta) g_{os}} \left[ 1 + \frac{8}{5} \eta(3\eta - 2) \alpha_s g_{os} \right]
    \]
Constitutive equations: bulk viscosity

- Bulk viscosity accounts for resistance of solid body to dilatation:

$$\lambda_s = \frac{4}{3} \alpha_s \rho_s d_s g_{os} (1 + e_s) \left( \frac{\theta_s}{\pi} \right)^{\frac{1}{2}}$$

- $\alpha_s$ volume fraction of solid.
- $e_s$ coefficient of restitution.
- $d_s$ particle diameter.
Plastic regime: frictional viscosity

- In the limit of maximum packing the granular flow regime becomes incompressible. The solid pressure decouples from the volume fraction.
- In frictional flow, the particles are in enduring contact and momentum transfer is through friction. The stresses are determined from soil mechanics (Schaeffer, 1987).
- The frictional viscosity is:
  \[ \mu_{s,\text{frict}} = \frac{P_s \sin \phi}{2\sqrt{I_2}} \]
- The effective viscosity in the granular phase is determined from the maximum of the frictional and shear viscosities:
  \[ \mu_s = \max \{ \mu_{s,\text{coll}} + \mu_{s,\text{kin}}, \mu_{s,\text{frict}} \} \]
Momentum equation: interphase forces

- Interaction between phases.
  \[ \sum_{s=1}^{n} (\vec{R}_{fs} + \dot{m}_{fs} \vec{u}_{fs}) = 0 \]

- Formulation is based on forces on a single particle corrected for effects such as concentration, clustering particle shape and mass transfer effects. The sum of all forces vanishes.
  - Drag: caused by relative motion between phases; \( K_{fs} \) is the drag between fluid and solid; \( K_{ls} \) is the drag between particles
    \[ \sum_{l=1}^{n} (K_{ls} (\vec{u}_l - \vec{u}_s)) + K_{fs} (\vec{u}_f - \vec{u}_s) = 0 \]

  General form for the drag term:
  \[ K_{fs} = \alpha_s \rho_s \frac{f_{drag}}{\tau_{fs}} \]

  With particle relaxation time:
  \[ \tau_{fs} = \frac{\rho_s d_s^2}{18 \mu_f} \]
Momentum: interphase exchange models

- Fluid-solid momentum interaction, expressions for $f_{\text{drag}}$.
  - Di Felice (1994).
  - Wen and Yu (1966).
- Drag based on Richardson and Zaki (1954) and/or Ergun (1952).
  - use the one that correctly predicts the terminal velocity in dilute flow.
  - in bubbling beds ensure that the minimum fluidized velocity is correct.
  - It depends strongly on the particle diameter: correct diameter for non-spherical particles and/or to include clustering effects.
Comparison of drag laws

• A comparison of the fluid-solid momentum interaction, $f_{\text{drag}}$, for:
  – Relative Reynolds number of 1 and 1000.
  – Particle diameter 0.001 mm.
Particle-particle drag law

- Solid-solid momentum interaction.
  - Drag function derived from kinetic theory (Syamlal et al, 1993).

\[
K_{lm} = \frac{3(1 + e_{lm})(\frac{\pi}{2} + C_{lm} \frac{\pi^2}{8})\alpha_l \rho_l \alpha_m \rho_m (d_l + d_m)^2}{2\pi(\rho_l d_l^3 + \rho_m d_m^3)} \frac{g_{olm}}{|\bar{u}_l - \bar{u}_m|}
\]

\[
g_{olm} = \frac{1}{\alpha_f} + \frac{3d_m d_l}{\alpha_f^2 (d_l + d_m)} \sum_{k=1}^{M} \frac{\alpha_k}{d_k}
\]
Momentum: interphase exchange models

- Virtual mass effect: caused by relative acceleration between phases Drew and Lahey (1990).

\[ K_{vm,fs} = C_{vm} \alpha_s \rho_f \left( \frac{\partial \tilde{u}_f}{\partial t} + \tilde{u}_f \cdot \nabla \tilde{u}_f \right) - \left( \frac{\partial \tilde{u}_s}{\partial t} + \tilde{u}_s \cdot \nabla \tilde{u}_s \right) \]


\[ K_{k,fs} = C_L \alpha_s \rho_f \left( \tilde{u}_f - \tilde{u}_s \right) \times \left( \nabla \times \tilde{u}_f \right) \]

- Other interphase forces are: Basset Force, Magnus Force, Thermophoretic Force, Density Gradient Force.
Granular multiphase model: mass transfer

- Unidirectional mass transfer: $\dot{m}_{fs}$

- Defines positive mass flow is specified constant rate of rate per unit volume from phase f to phase s,
  - $\dot{m}_{fs}$ proportional to: $\dot{r}\alpha_f \rho_s$
  - particle shrinking or swelling.
    - e.g., rate of burning of particle.
  - Heat transfer modeling can be included via UDS.
Granular temperature equations

- Granular temperature. \( \theta = \frac{1}{3} \vec{C} \cdot \vec{C} \)

\[
\frac{3}{2} \left\{ \frac{\partial}{\partial t} (\alpha_s \rho_s \theta_s) + \nabla \cdot (\alpha_s \rho_s \vec{u}_s \theta_s) \right\} = \tau_s : \nabla \vec{u}_s + \ldots
\]

- Production term

- Diffusion term

- Dissipation term due to inelastic collisions

- Exchange terms
Constitutive equations: granular temperature

- Granular temperature for the solid phase is proportional to the kinetic energy of the random motion of the particles.

  $\bar{\tau}_s : \nabla \bar{u}_s$

  represents the generation of energy by the solids stress tensor.

- $\nabla \cdot (\kappa_{\theta s} \nabla \theta_s)$

  represents the diffusion of energy.

- $K_{\theta s}$

  Granular temperature conductivity.
Constitutive equations: granular temperature

- Granular temperature conductivity.
  - Syamlal:
    \[
    \kappa_{\theta} = \frac{15\alpha_s \rho_s d_s \sqrt{\theta_s \pi}}{4(41 - 33\eta)} \left[1 + \frac{12}{5} \alpha_s g_{os} \eta^2 (4\eta - 3) + \frac{16}{15\pi} (41 - 33\eta)\eta \alpha_s g_{os}\right]
    \]
  - Gidaspow:
    \[
    \kappa_{\theta} = \frac{75 \rho_s d_s \sqrt{\theta_s \pi}}{384\eta g_{os}} \left[1 + \frac{12}{5} \alpha_s g_{os} \eta \right]^2 + 2\alpha_s^2 \rho_s d_s (1 + e_s) g_{os} \sqrt{\frac{\theta_s}{\pi}}
    \]
  - Sinclair:
    \[
    \kappa_{\theta} = (\kappa_{\theta})_{syamlal} + \frac{25 \rho_s d_s \sqrt{\theta_s \pi}}{16\eta g_{os}} \omega \left[1 + \frac{12}{5} \eta^2 (4\eta - 3)\alpha_s g_{os} \right]
    \]
Constitutive equations: granular temperature

- $\gamma_{\theta_s}$ represents the dissipation of energy due to inelastic collisions.
  - Gidaspow: $\gamma_{\theta_s} = 3(1-e_s^2)\alpha_s^2 \rho_s g_{os} \theta_s \left[ \frac{4}{\pi} \sqrt{\frac{\theta}{\pi}} - \nabla \bar{u}_s \right]
  
  - Syamlal and Sinclair: $\gamma_{\theta_s} = \frac{12(1-e_s)g_{os}}{d_s \sqrt{\pi}} \rho_s \alpha_s \theta_s^{3/2}$ Lun et al (1984)

- Here $\phi_{lm}$ represents the energy exchange among solid phases (UDS).
Constitutive equations: granular temperature

- $\phi_{fs}$ represents the energy exchange between the fluid and the solid phase.
  - Laminar flows: $\phi_{fs} = -3K_{fs}\theta_s$
  
  - Dispersed turbulent flows:
    - Sinclair: $\phi_{fs} = K_{fs}(\sqrt{2k_f}\sqrt{3\theta} - 2k_f)$
    - Other models: $\phi_{fs} = K_{fs}(2k_f - \langle u_{pi}', u_{fi}' \rangle)$
Test case for Eulerian granular model

- Contours of solid stream function and solid volume fraction when solving with Eulerian-Eulerian model.

- Contours of solid stream function and solid volume fraction when solving with Eulerian-Granular model.
Solution guidelines

- All multiphase calculations:
  - Start with a single-phase calculation to establish broad flow patterns.

- Eulerian multiphase calculations:
  - Copy primary phase velocities to secondary phases.
  - Patch secondary volume fraction(s) as an initial condition.
  - For a single outflow, use OUTLET rather than PRESSURE-INLET; for multiple outflow boundaries, must use PRESSURE-INLET.
  - For circulating fluidized beds, avoid symmetry planes (they promote unphysical cluster formation).
  - Set the “false time step for underrelaxation” to 0.001.
  - Set normalizing density equal to physical density.
  - Compute a transient solution.
Summary

• The Eulerian-granular multiphase model has been described in the section.
• Separate flow fields for each phase are solved and the interaction between the phases modeled through drag and other terms.
• The Eulerian-granular multiphase model is applicable to all particle relaxation time scales and includes heat and mass exchange between phases.
• Several kinetic theory formulations available:
  • Gidaspow: good for dense fluidized bed applications.
  • Syamlal: good for a wide range of applications.
  • Sinclair: good for dilute and dense pneumatic transport lines and risers.