

Lecture 18 - Eulerian Flow Modeling

Applied Computational Fluid Dynamics

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Contents

- Overview of Eulerian-Eulerian multiphase model.
- Spatial averaging.
- Conservation equations and constitutive laws.
- Interphase forces.
- Heat and mass transfer.
- Discretization.
- Solver basics.

Eulerian-Eulerian multiphase - overview

- Used to model droplets or bubbles of secondary phase(s) dispersed in continuous fluid phase (primary phase).
- Allows for mixing and separation of phases.
- Solves momentum, enthalpy, and continuity equations for each phase and tracks volume fractions.
- Uses a single pressure field for all phases.
- Uses interphase drag coefficient.
- Allows for virtual mass effect and lift forces.
- Multiple species and homogeneous reactions in each phase.
- Allows for heat and mass transfer between phases.
- Can solve turbulence equations for each phase.

Methodology

- A general multiphase system consists of interacting phases dispersed randomly in space and time. Interpenetrating continua.
- Methods:
 - Use of continuum theory and thermodynamical principles to formulate the constitutive equations (ASMM).
 - Use of microstructural model in which macroscopic behavior is inferred from particle interaction: Eulerian-Granular.
 - Use of averaging techniques and closure assumptions to model the unknown quantities:
 - Space averaging with no time averaging.
 - Time averaging with no space averaging.
 - Ensemble averaging with no space averaging.
 - space/time or ensemble/space averaging: Eulerian-Eulerian.

Two-fluid model (interpenetrating continua)

- Deductive approach:
 - Assume equations for each pure phase.
 - Average (homogenize) these equations.
 - Model the closure terms.
- Inductive approach:
 - Assume equations for interacting phases.
 - Model the closure terms.

Spatial averaging: basic equations

- Application of the general transport theorem to a property Ψ_k gives the general balance law and its jump condition:

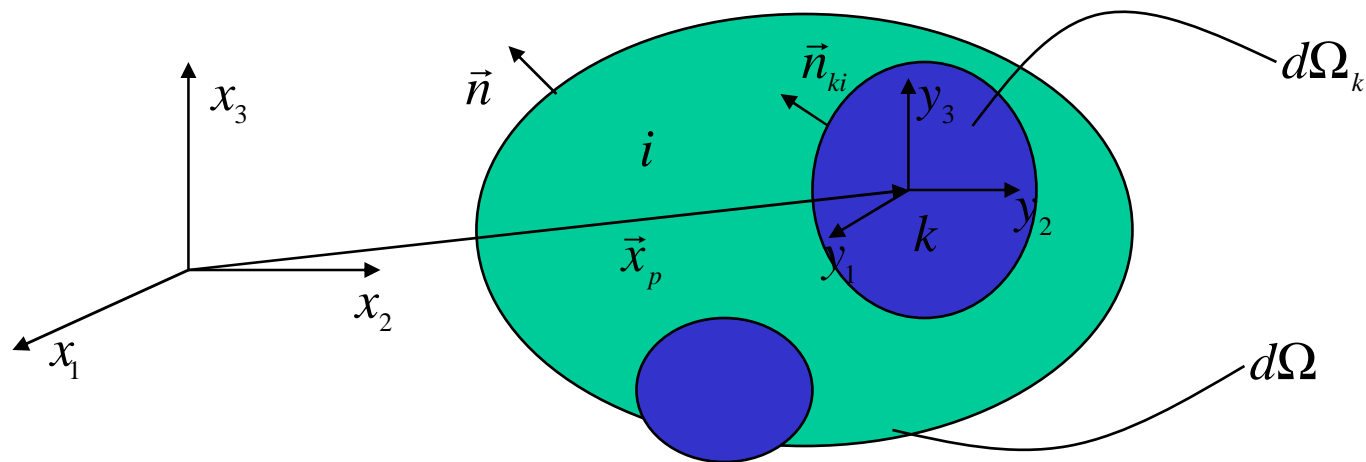
$$\frac{\partial \psi_k}{\partial t} + \nabla \cdot \vec{u}_k \psi_k + \nabla \cdot \vec{J}_k = \dot{\phi}_k \quad \sum_{i=1, k \neq i}^n (\psi_k (\vec{u}_k - \vec{u}_i) \cdot \vec{n}_{ki} + n_{ki} \cdot \vec{J}_k) = 0$$

- Continuity equation: $\psi_k = \rho_k, \vec{J}_k = 0, \dot{\phi}_k = 0$

- Momentum equation: $\psi_k = \rho_k \vec{u}_k, \vec{J}_k = P_k \vec{I} - \vec{\tau}_k, \dot{\phi}_k = \rho_k \vec{F}_k$

Spatial averaging

- Consider an elementary control volume $d\Omega$ bounded by the surface dS .
 - Length scales: $L \gg l \gg d_k \sim l_t$
 - Volumes: $d\Omega = \sum_{i=1}^n d\Omega_i(t)$
 - Averaging volume and coordinate system:



Space averaging: basic equations

- Definitions:

- Volume average

$$\langle f \rangle = \frac{1}{d\Omega} \int_{d\Omega_i} f(\vec{x} + \vec{y}, t) d'\Omega$$

Intrinsic phasic average.

$$\langle f \rangle_k = \frac{1}{d\Omega_k} \int_{d\Omega_k} f(\vec{x} + \vec{y}, t) d'\Omega$$

- Volume phase fraction:

$$\alpha_k = \frac{d\Omega_k(\vec{x}_p, t)}{d\Omega}$$

$$\sum_{k=1}^n \alpha_k = 1$$

$$\langle f \rangle = \alpha_k \langle f \rangle_k$$

Space averaging: averaging theorems

- For all k-volumes that are differentiable, (Gray and Lee (1977), Howes and Whitaker (1985)).

- Temporal derivative: $\langle \frac{\partial f}{\partial t} \rangle \neq \frac{\partial}{\partial t} \langle f \rangle$

$$\langle \frac{\partial f}{\partial t} \rangle = \frac{\partial}{\partial t} \langle f \rangle - \frac{1}{d\Omega} \int_{dS_{ki}} f \vec{u}_{ki} \cdot \vec{n}_{ki} d'S$$

- Spatial derivative: $\langle \nabla f \rangle \neq \nabla \langle f \rangle$


$$\langle \nabla f \rangle = \nabla \langle f \rangle + \frac{1}{d\Omega} \int_{dS_{ki}} f \vec{n}_{ki} d'S$$

- Next, apply averaging to the conservation equations.

Space averaging: conservation equations

- Continuity equation:

$$\frac{\partial}{\partial t} \alpha_k \langle \rho_k \rangle_k + \nabla \cdot \alpha_k \langle \rho_k \vec{u}_k \rangle_k = - \frac{1}{d\Omega} \int_{dS_{ki}} \rho_k (\vec{u}_k - \vec{u}_{ki}) \cdot \vec{n}_k d'S$$

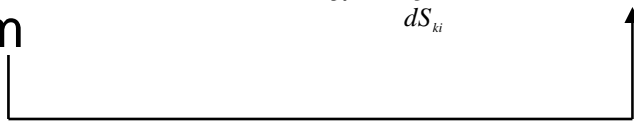


 mass transfer term

- Momentum equation:

$$\frac{\partial}{\partial t} \alpha_k \langle \rho_k \vec{u}_k \rangle_k + \nabla \cdot \alpha_k \langle \rho_k \vec{u}_k \vec{u}_k \rangle_k = - \nabla \alpha_k \langle P_k \rangle_k + \nabla \cdot \alpha_k \langle \vec{\tau}_k \rangle_k + \alpha_k \langle \rho_k F_k \rangle_k$$

$$+ \frac{1}{d\Omega} \int_{dS_{ki}} \{ \rho_k \vec{u}_k (\vec{u}_{ki} - \vec{u}_k) - P_k + \vec{\tau}_k \} \cdot \vec{n}_{ki} d'S$$



 interaction term

Interphase forces

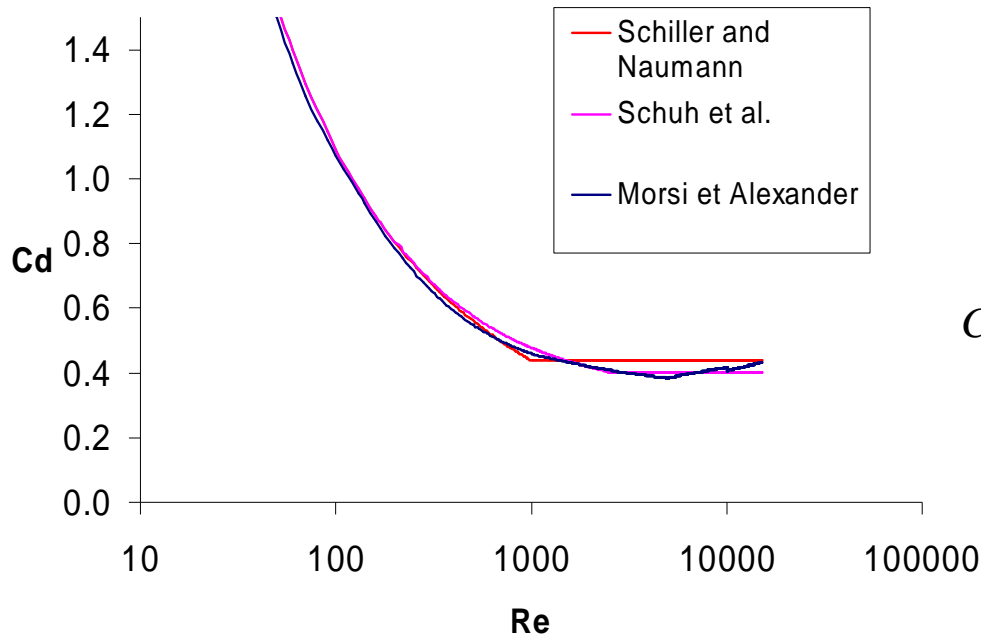
- Drag is caused by relative motion between phases.

$$\sum_{i=1}^n K_{ik} (\vec{u}_i - \vec{u}_k) = 0, \quad K_{ik} = \alpha_k \rho_k \frac{f_{drag}}{\tau_{ik}}, \quad \tau_{ik} = \frac{\rho_k d_k^2}{18\mu_i}$$

- Commonly used drag models (fluid-fluid multiphase).
 - Schuh et al. (1989).
 - Schiller and Naumann (1935).
 - Morsi and Alexander (1972).
 - Schwarz and Turner (1988; for bubble columns).
 - Symmetric law.
- Many researchers devise and implement their own drag models for their specific systems.

Interphase forces: drag force models

Fluid-fluid drag functions



Schiller and Naumann

$$C_D = \begin{cases} 24(1+0.15\text{Re}^{0.687}) & \text{Re} \leq 1000 \\ 0.44 & \text{Re} > 1000 \end{cases}$$

Schuh et al.

$$C_D = \begin{cases} 24(1+0.15\text{Re}^{0.687}) & 0 < \text{Re} \leq 200 \\ 24(0.914\text{Re}^{0.282} + 0.0135\text{Re})/\text{Re} & 200 \leq \text{Re} < 2500 \\ 0.4008 & \text{Re} > 2500 \end{cases}$$

Morsi and Alexander

$$C_D = a_1 + \frac{a_2}{\text{Re}} + \frac{a_3}{\text{Re}^2} \quad \text{where } a_1, a_2, a_3 \text{ are } f(\text{Re})$$

Interphase forces: virtual mass and lift

- Virtual mass effect: caused by relative acceleration between phases Drew and Lahey (1990).

$$K_{vm,fs} = C_{vm} \alpha_s \rho_f \left(\left(\frac{\partial \vec{u}_f}{\partial t} + \vec{u}_f \cdot \nabla \vec{u}_f \right) - \left(\frac{\partial \vec{u}_s}{\partial t} + \vec{u}_s \cdot \nabla \vec{u}_s \right) \right)$$

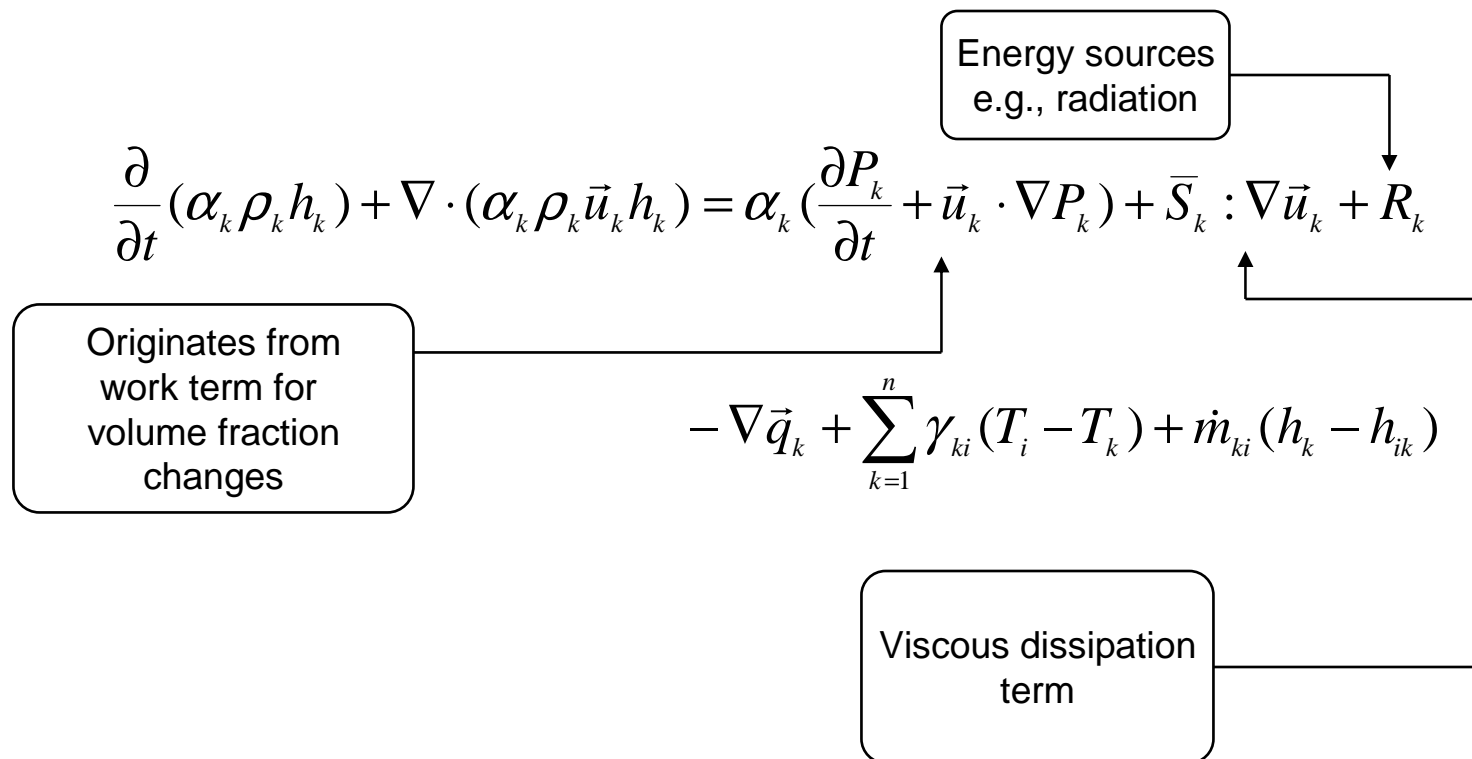
- Virtual mass effect is significant when the second phase density is much smaller than the primary phase density (i.e., bubble column).
- Lift force: caused by the shearing effect of the fluid onto the particle Drew and Lahey (1990).

$$K_{k,fs} = C_L \alpha_s \rho_f (\vec{u}_f - \vec{u}_s) \times (\nabla \times \vec{u}_f)$$

- Lift force usually insignificant compared to drag force except when the phases separate quickly and near boundaries.

Modeling heat transfer

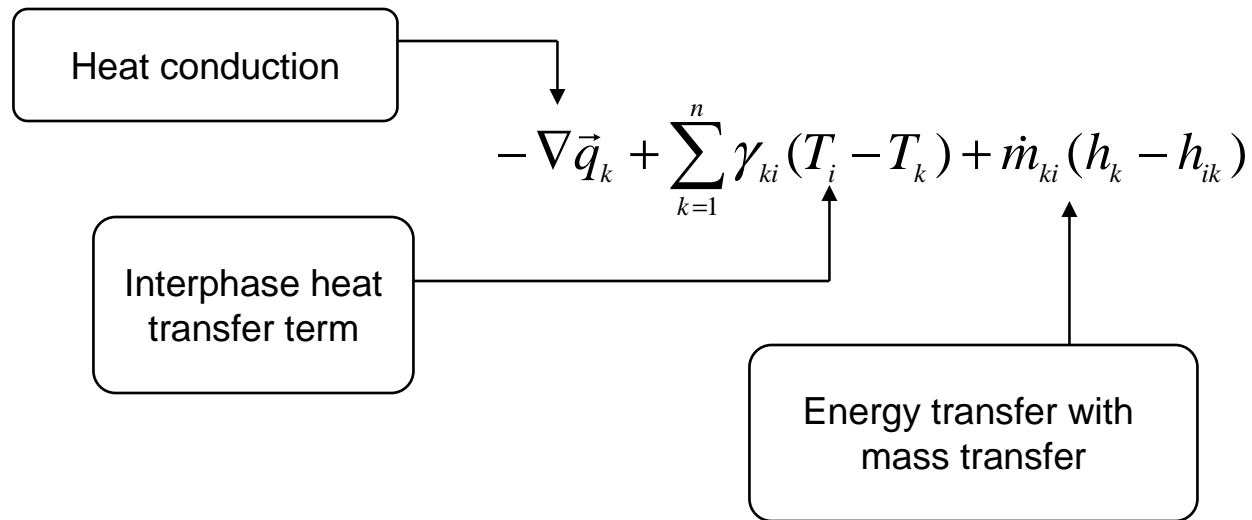
- Conservation equation of phase enthalpy.



Modeling heat transfer

- Conservation equation of phase enthalpy.

$$\frac{\partial}{\partial t}(\alpha_k \rho_k h_k) + \nabla \cdot (\alpha_k \rho_k \vec{u}_k h_k) = \alpha_k \left(\frac{\partial P_k}{\partial t} + \vec{u}_k \cdot \nabla P_k \right) + \bar{S}_k : \nabla \vec{u}_k + R_k$$



Heat conduction

- Assume Fourier's law:

$$\vec{q} = -\alpha_k \kappa_k \nabla T_k$$

For granular flows κ_k is obtained from packed bed conductivity expression, Kuipers, Prins and Swaaij (1992).

- Near wall heat transfer is calculated as in single phase.
- All standard boundary conditions for temperature can be implemented for multiphase.

Interphase heat transfer

- The interphase heat transfer coefficient is given by.

$$\gamma_{ik} = \frac{6\kappa_k \alpha_i Nu_i}{d_i^2}$$

- Granular Model (Gunn, 1978).

$$Nu_s = (7 - 10\alpha_f + 5\alpha_f^2)(1 + 0.7 Re^{0.2} Pr^{1/3}) \\ + (1.33 - 2.4\alpha_f + 1.2\alpha_f^2) Re^{0.7} Pr^{1/3}$$

- Fluid-fluid model.

$$Nu_i = 2 + 0.6 Re^{0.5} Pr^{0.3}$$

$$Re = \frac{\alpha_k \rho_k |u_i - u_k| d_i}{\mu_k}$$

$$Pr = \frac{C_{p,k} \mu_k}{\kappa_k}$$

Conservation of species

- Conservation equation for the mass fraction m_k^i of the species i in the phase k :

$$\frac{\partial}{\partial t}(\alpha_k \rho_k m_k^i) + \nabla \cdot (\alpha_k \rho_k \vec{u}_k m_k^i) = \nabla \cdot (\alpha_k \rho_k D_k^i \nabla m_k^i) + \dot{m}_k^i$$

- Here D_k^i is the diffusivity of the species in the mixture of the respective phase, \dot{m}_k^i is the rate of production/destruction of the species.
- Thermodynamic relations and state equations for the phase k are needed.
- When calculating mass transfer the shadow technique from Spalding is used to update diameter of the dispersed phase.

Modeling mass transfer

- Evaporation and condensation:

- For liquid temperatures \geq saturation temperature, evaporation rate:

$$\dot{m}_v = \frac{r_v \alpha_l \rho_l (T_l - T_{sat})}{T_{sat}}$$

- For vapor temperatures \leq saturation temperature, condensation rate:

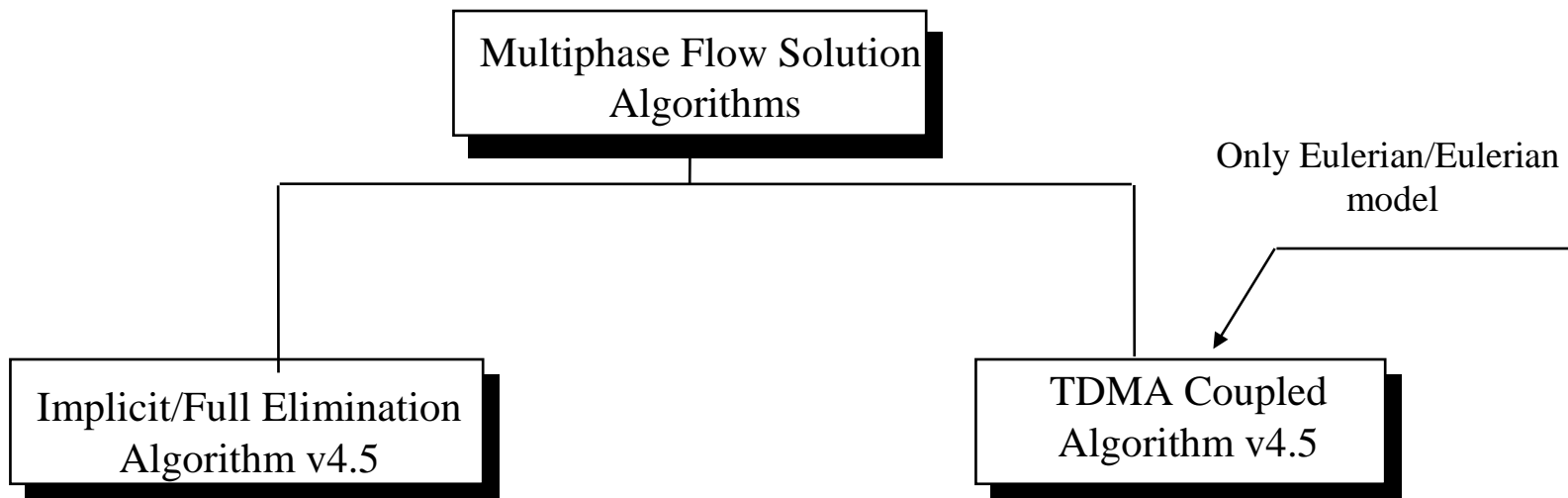
$$\dot{m}_l = \frac{r_l \alpha_v \rho_v (T_{sat} - T_v)}{T_{sat}}$$

- User specifies saturation temperature and, if desired, “time relaxation parameters” r_l and r_v (Wen Ho Lee (1979)).

- Unidirectional mass transfer, r is constant: $\dot{m}_{12} = r \alpha_2 \rho_1$

Solution algorithms for multiphase flows

- Coupled solver algorithms (more coupling between phases).
 - Faster turn around and more stable numerics.
- High order discretization schemes for all phases.
 - More accurate results.



Full elimination algorithm

- Momentum equation for primary and secondary phase:

$$(a_p + k)u_p - k u_s = \sum a_{nb,p} u_{nb,p} + b_p$$

$$-k u_p + (a_s + k)u_s = \sum a_{nb,s} u_{nb,s} + b_s$$

- Elimination of secondary phase gives primary phase:

$$\left(a_p + \frac{k}{a_s + k} a_s \right) u_p = \sum a_{nb,p} u_{nb,p} + b_p + \frac{k}{a_s + k} \left(\sum a_{nb,s} u_{nb,s} + b_s \right)$$

$$a_{p,eff} u_p = \sum a_{nb,p} u_{nb,p} + b_p + b_{p,s}$$

- Secondary phase has similar form.
- Applicable to N phases.

Coupled TDMA-algorithm

- Discretized equations of primary and secondary phase are in matrix form:

$$\begin{pmatrix} a_p + k & -k \\ -k & a_s + k \end{pmatrix} \begin{pmatrix} u_p \\ u_s \end{pmatrix} = \begin{pmatrix} \sum a_{nb,p} & 0 \\ 0 & \sum a_{nb,s} \end{pmatrix} \begin{pmatrix} u_{nb,p} \\ u_{nb,s} \end{pmatrix} + \begin{pmatrix} b_p \\ b_s \end{pmatrix}$$

$$\overline{\overline{\mathbf{A}}} \vec{U} = \overline{\overline{\mathbf{A}}}_{nb} \vec{U}_{nb} + \vec{B}$$

- Results in a tri-diagonal matrix consisting of submatrices

$$\begin{pmatrix} A_{11} & A_{12} & 0 & \vdots & 0 \\ A_{21} & A_{22} & A_{23} & \vdots & 0 \\ 0 & A_{32} & A_{33} & \vdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \vdots & A_{nn} \end{pmatrix} \begin{pmatrix} \vec{U}_1 \\ \vec{U}_2 \\ \vec{U}_3 \\ \vdots \\ \vec{U}_n \end{pmatrix} = \begin{pmatrix} \vec{C}_1 \\ \vec{C}_2 \\ \vec{C}_3 \\ \vdots \\ \vec{C}_n \end{pmatrix}$$

- Closer coupling in each iteration gives faster convergence

Solution algorithm for multiphase

- Typical algorithm:
 - Get initial and boundary conditions.
 - Perform time-step iteration.
 - Calculate primary and secondary phase velocities.
 - Calculate pressure correction and correct phase velocities, pressure and phase fluxes. Pressure is shared by all phases.
 - Calculate volume fraction.
 - Calculate other scalars. If not converged go to step three.
 - Advance time step and go to step two.

Solution guidelines

- All multiphase calculations:
 - Start with a single-phase calculation to establish broad flow patterns.
- Eulerian multiphase calculations:
 - Copy primary phase velocities to secondary phases.
 - Patch secondary volume fraction(s) as an initial condition.
 - Set normalizing density equal to physical density.
 - Compute a transient solution.
 - Use multigrid for pressure.

Summary

- Eulerian-Eulerian is the most general multiphase flow model.
- Separate flow field for each phase.
- Applicable to all particle relaxation time scales.
- Includes heat/mass exchange between phases.
- Available in both structured and unstructured formulations.