Blend Times in Stirred Tanks

Reacting Flows - Lecture 9

Instructor: André Bakker

Evaluation of mixing performance

- Methods to evaluate mixing performance:
  - Characterization of homogeneity.
  - Blending time.
- General methods to characterize homogeneity:
  - Visual uniformity.
  - Quantitative change in local concentration as a function of time.
  - Review instantaneous statistics about the spatial distribution of the species.
    - Average concentration
    - Minimum and maximum
    - Standard deviation in the concentration.
    - Coefficient of variation CoV = standard deviation/average.

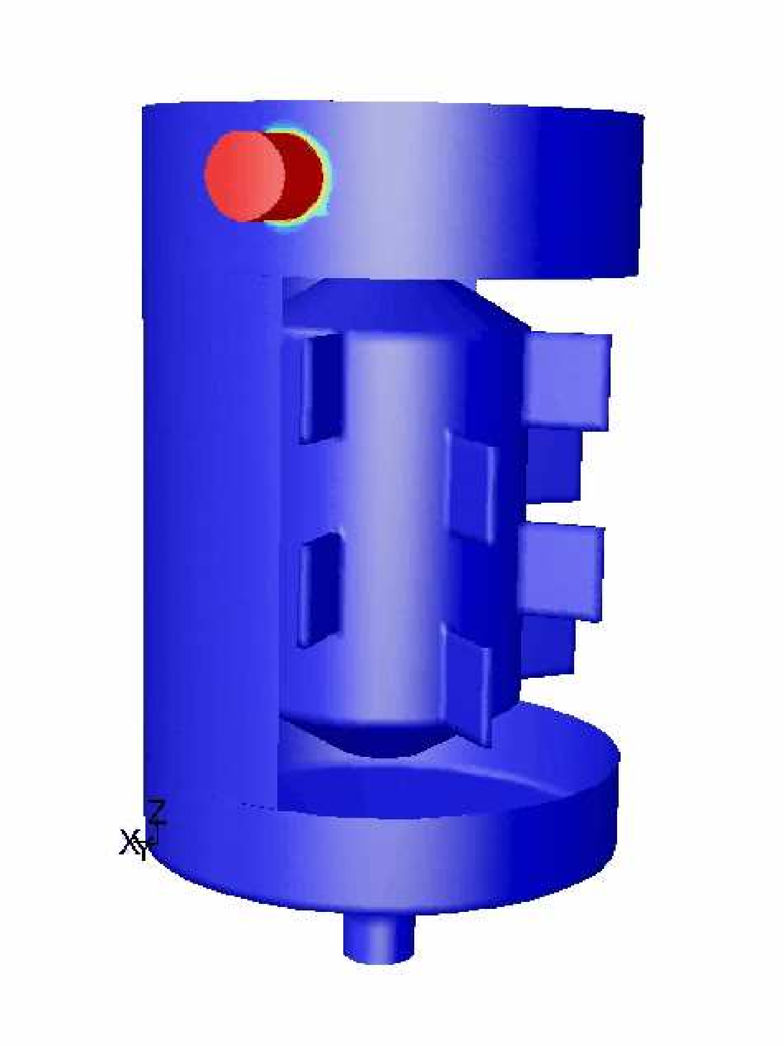
Visual uniformity

- Experimentally measure the time it takes to obtain visual uniformity.
- Can be done with acid-base additions and a pH indicator.
- Offers good comparisons between performance of different mixing systems.
- Not a suitable approach for CFD.



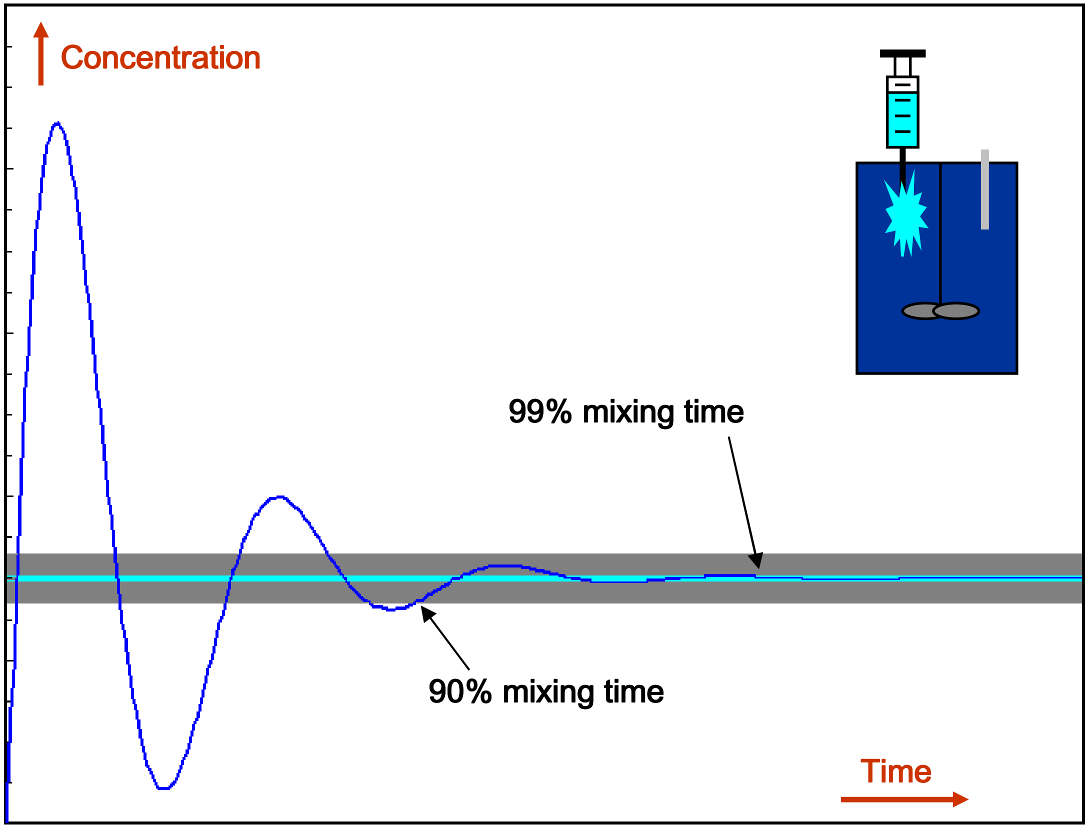

Visual uniformity example: glass mixing

- Glass exits the glass ovens with variations in temperature and material concentrations.
- As a result, when the glass hardens, there will be visual non-uniformities.
- So, glass needs to be mixed before it is used. Because of the high-viscosity and temperature, special mixers are used,
- Optical quality of glass is still often determined visually.



Quantitative variation in a point

- Measuring the tracer concentration as a function of time c(t) in one or more points in the vessel, is a common experimental method.
- The mixing time is then the time it takes for the measured concentration c(t) to stay within a certain range of the final concentration c_∞.
- Advantage: easy to use in experiments.
- Disadvantage: uses only one or a few points in the vessel.
- Does not use all information present in a CFD simulation.



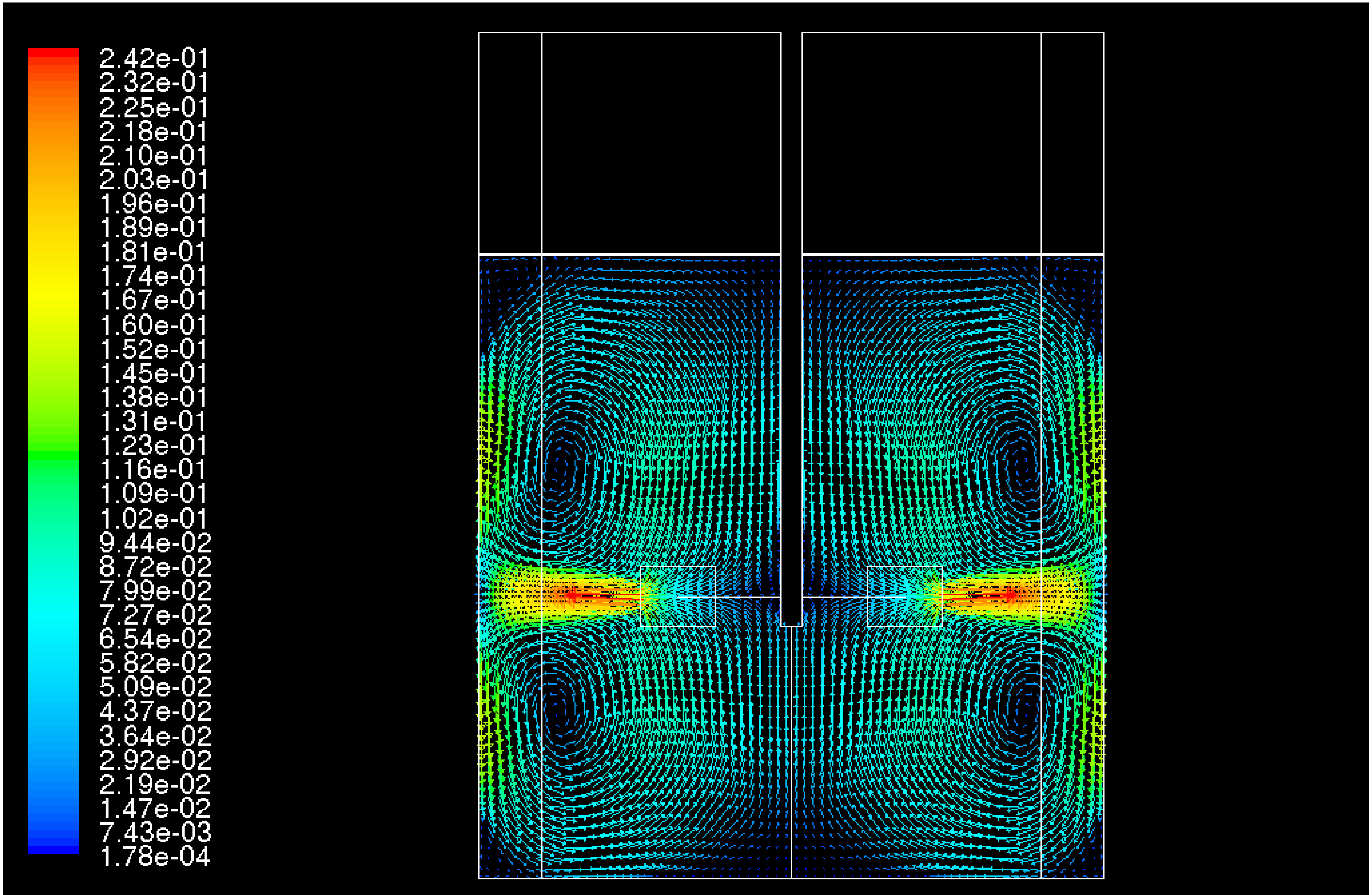
Blend time calculations with CFD

- Transport and mixing of a tracer:
  - Add tracer to the domain.
  - Mass fraction of tracer calculated and monitored as a function of time.
  - Determine blend time based on the mass-fraction field satisfying a pre-specified criterion.
- Flow field required can be steady, frozen unsteady or unsteady.
- Benefit of CFD:
  - The full concentration field is known.
  - Can use more data to determine blend time than what can be measured experimentally using probes.
- Main question: what should be the mixing criterion?

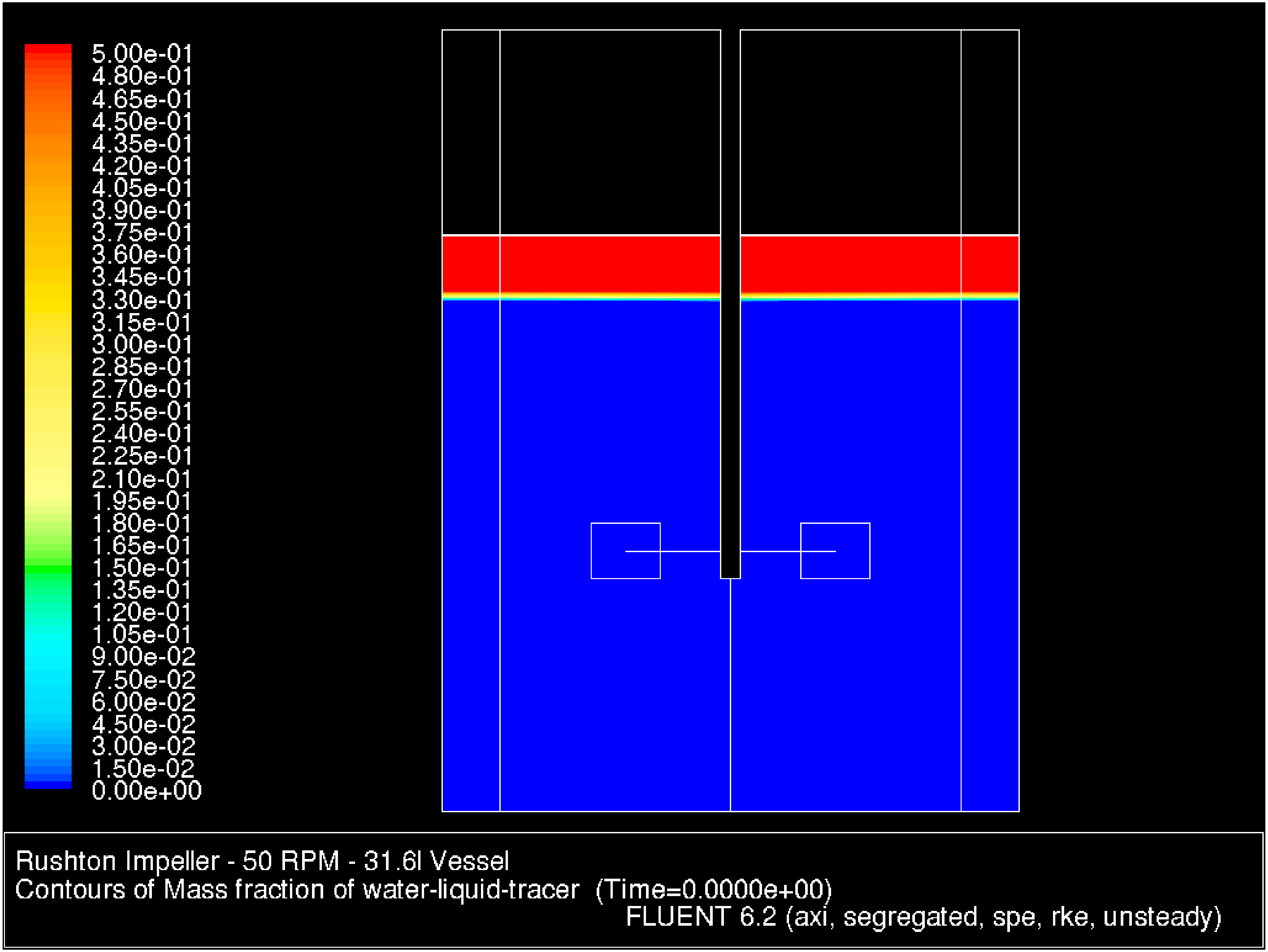
CFD analysis for blend time

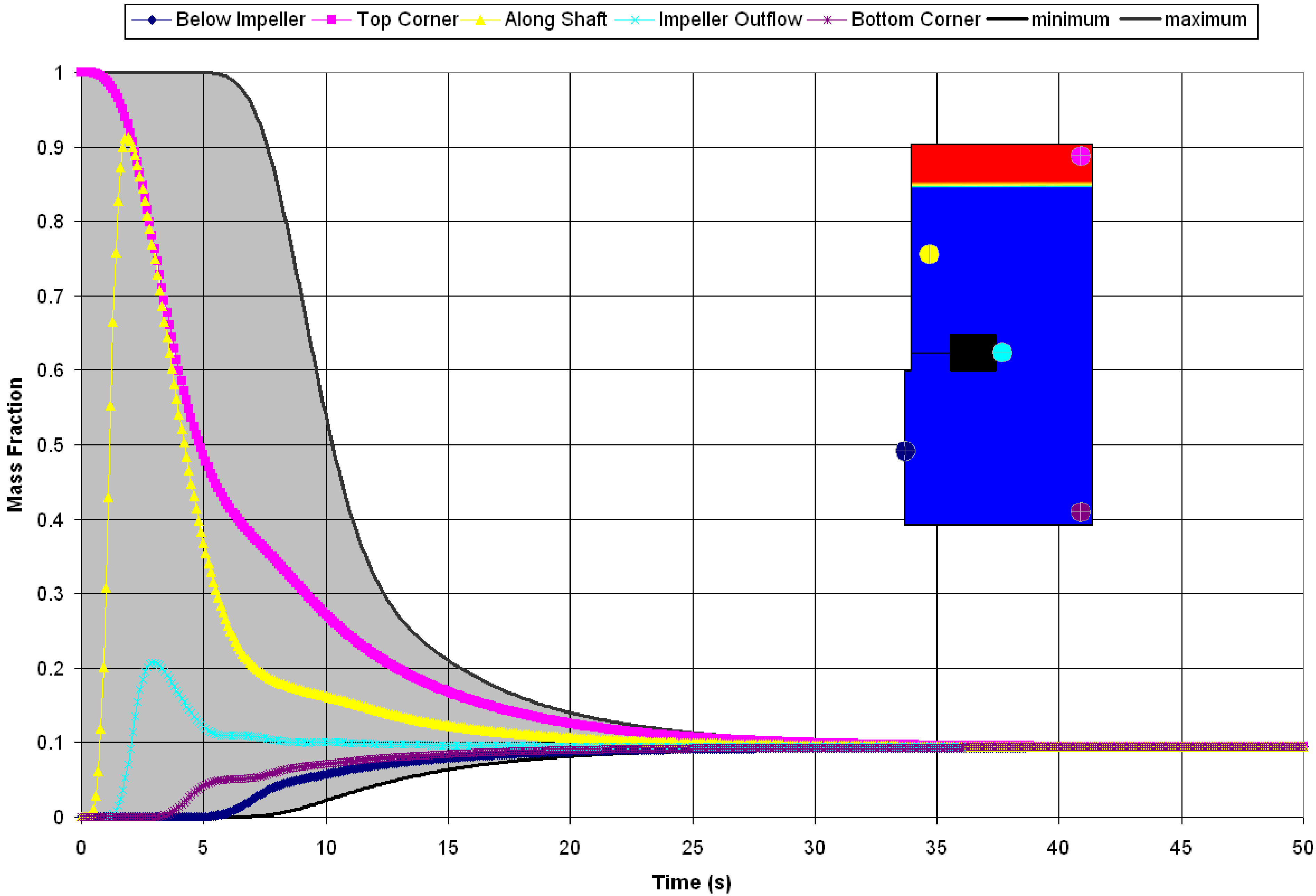
		Physical Lab	*CFD Lab*
Addition of Tracer	Volume of Tracer	Controlled	Exact
	Delivery Time	finite	zero
	Location	variable	fixed
Concentration Measurement of Tracer	Conductivity	Yes	No
	Color	Yes	No
	Mass Fraction	inferred	Yes

- We will now:
  - Illustrate the blend time analysis using a 2-D Rushton turbine flow field example.
  - Tracer added and its transport and mixing calculated. Mass fractions are monitored as a function of time.
  - Blend time is calculated using different criteria.



Rushton Impeller - 50 RPM - 31.6l Vessel
Velocity Vectors Colored By Velocity Magnitude (m/s)


5.00e-01
4.80e-01
4.65e-01
4.50e-01
4.35e-01
4.20e-01
4.05e-01
3.90e-01
3.75e-01
3.60e-01
3.45e-01
3.30e-01
3.15e-01
3.00e-01
2.85e-01
2.70e-01
2.55e-01
2.40e-01
2.25e-01
2.10e-01
1.95e-01
1.80e-01
1.65e-01
1.50e-01
1.35e-01
1.20e-01
1.05e-01
9.00e-02
7.50e-02
6.00e-02
4.50e-02
3.00e-02
1.50e-02
0.00e+00
Rushton Impeller - 50 RPM - 31.6l Vessel
Contours of Mass fraction of water-liquid-tracer (Time=0.0000e+00)
FLUENT 6.2 (axi, segregated, spe, rke, unsteady)



Below Impeller
Top Corner
Along Shaft
Impeller Outflow
Bottom Corner
minimum
maximum
Mass Fraction
Time (s)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
5
10
15
20
25
30
35
40
45
50

Measures of variation

- Variations in Y, the mass fraction of tracer, can be measured in several ways. For all measures, greater numbers indicate a greater variation with no upper bound.

- Coefficient of variation. Ratio between standard deviation σ_Y and the average $<Y>$:

$$CoV = \frac{\sigma_Y}{<Y>}$$

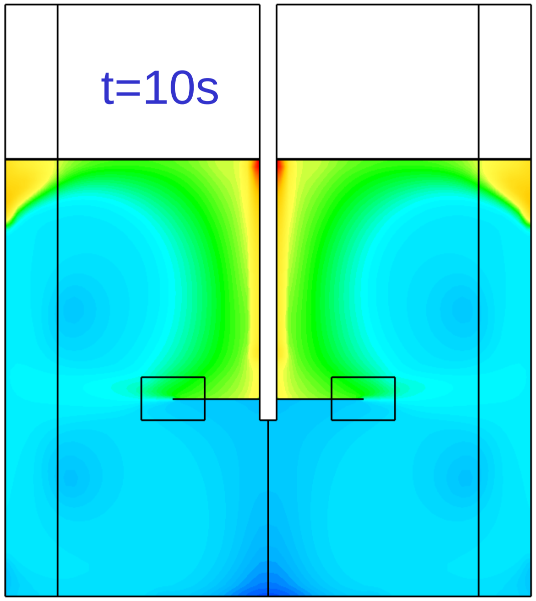
- Ratio between maximum and minimum mass fractions Y_{max}/Y_{min}.

- Largest deviation between extremes in the mass fraction and the average:

$$\Delta_{\max} = \max\left(Y_{\max} - <Y>,\ <Y> - Y_{\min}\right)$$

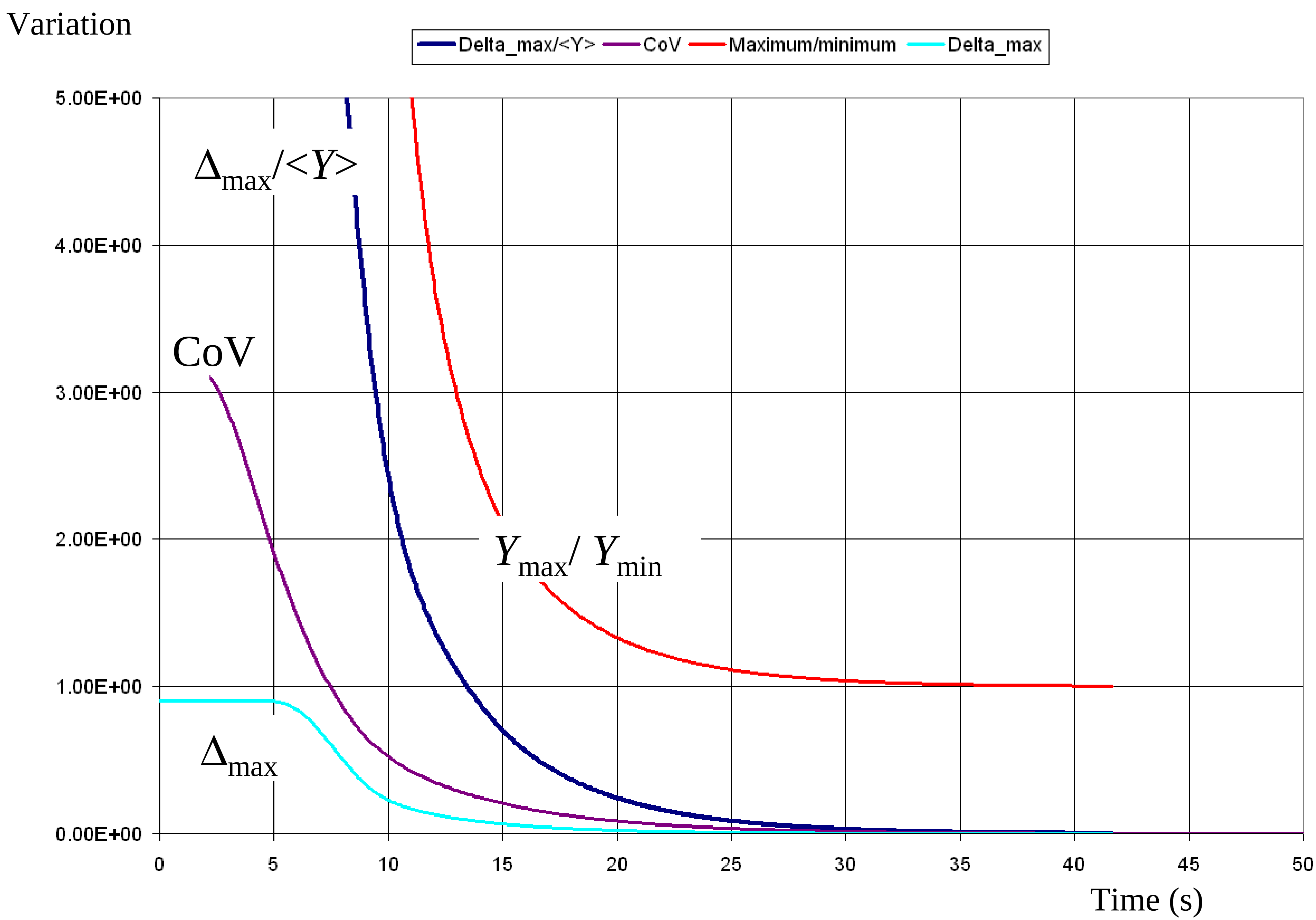
Can also be normalized over $<Y>$.

Variation calculation example



- Mass fraction data:
  - Min-max anywhere: 0.0223-0.539
  - Min-max from probes: 0.0574-0.272
  - Average: 0.0943
  - Standard deviation: 0.0493

- Measures of variation:
  - Max/min = 0.539/0.0223 = 24.2 (anywhere)
  - Max/min = 0.272/0.0574 = 4.7 (from probes)
  - CoV = 0.0493/0.0943 = 0.52
  - Δ_{max} = max(0.539-0.0943, 0.0943-0.0223) = 0.44
  - $\Delta_{max}/\langle Y \rangle$ = 0.44/0.0943 = 4.7



Variation
Delta_max/<Y>
CoV
Maximum/minimum
Delta_max
5.00E+00
4.00E+00
3.00E+00
2.00E+00
1.00E+00
0.00E+00
$\Delta_{max}/<Y>$
CoV
Y_{max}/ Y_{min}
Δ_{max}
0
5
10
15
20
25
30
35
40
45
50
Time (s)

Measures of uniformity - absolute

- There is a need to have an absolute measure of uniformity *U* that is ≤ 1 with 1 (or 100%) indicating perfect uniformity.

- Ratio between the minimum and maximum mass fractions.
  - Bounded between 0 and 1.

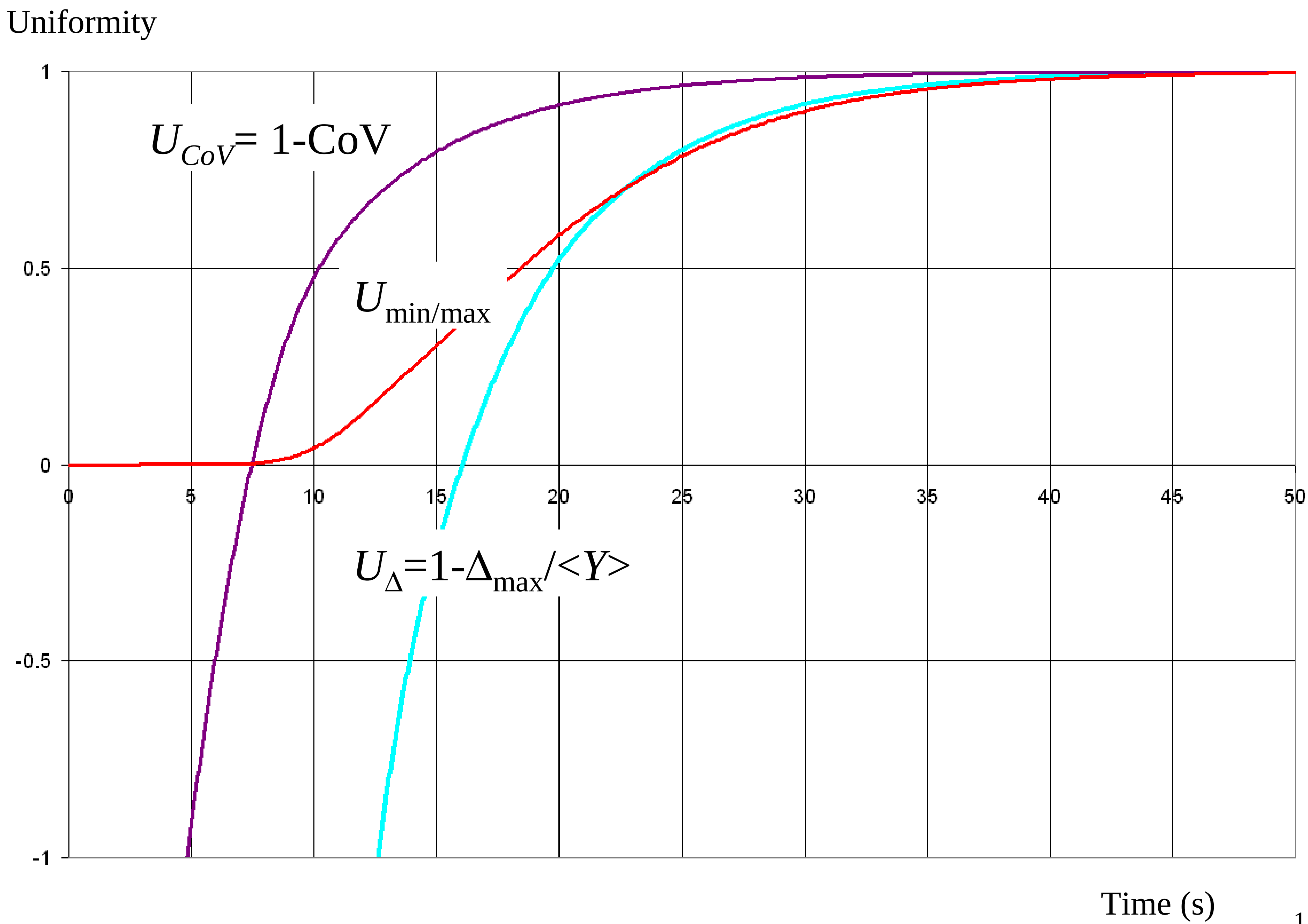
$$U_{\min/\max} = \frac{Y_{\min}}{Y_{\max}}$$

- Based on coefficient of variance CoV:
  - Not bounded: can be less than 0.

$$U_{CoV} = 1 - CoV$$

- Based on largest deviation from the average:
  - Conceptually closer to common experimental techniques.
  - Not bounded: can be less than 0.

$$U_{\Delta} = 1 - \frac{\Delta_{\max}}{< Y >}$$



Uniformity
1
0.5
0
-0.5
-1
0
5
10
15
20
25
30
35
40
45
50
U_{CoV}= 1-CoV
$U_{\text{min/max}}$
U_{Δ}=1-Δ_{max}/<Y>
Time (s)

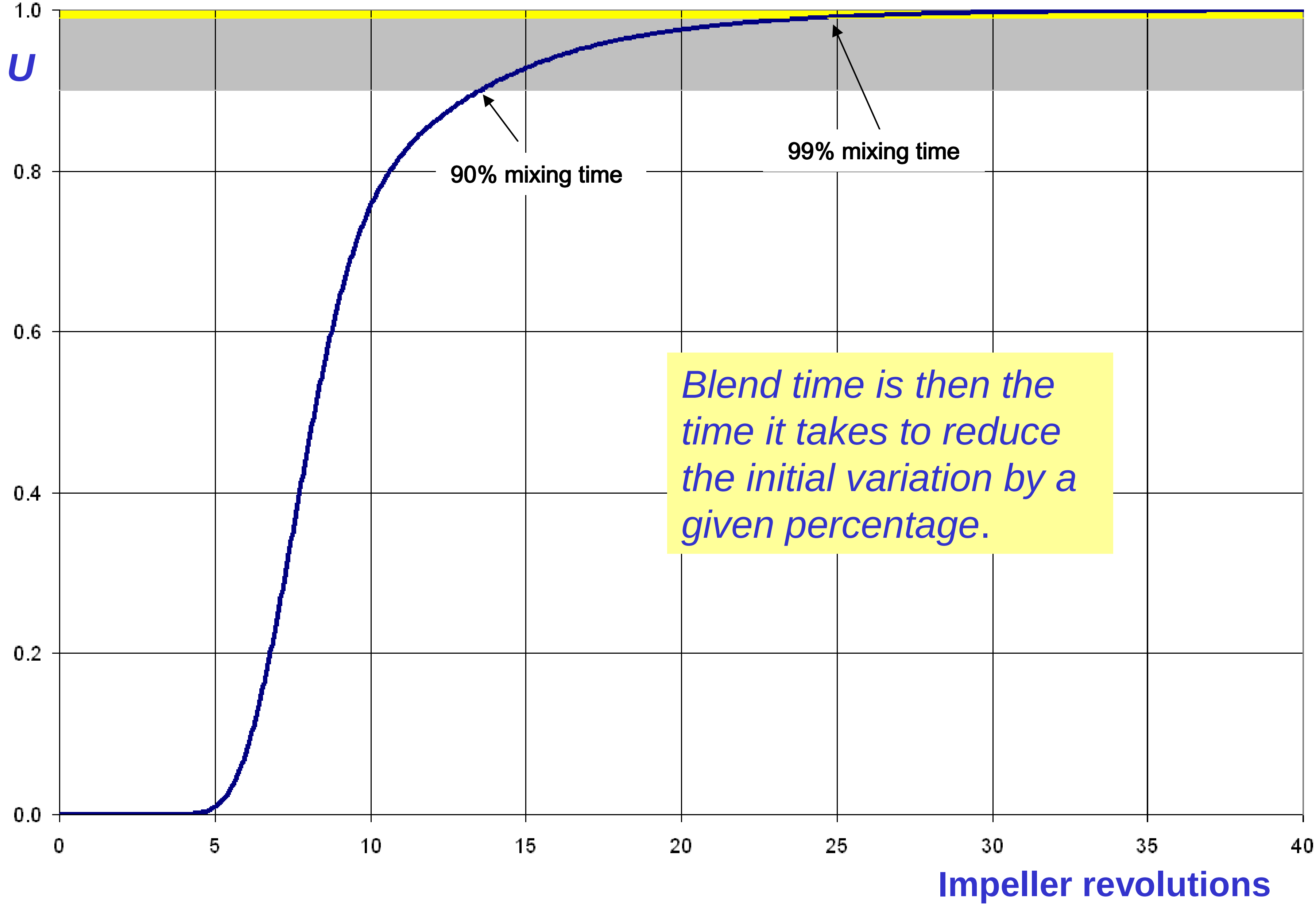
Uniformity

- These measures of uniformity:
  - All indicate perfect uniformity at values of 1.
  - Are not all bounded between 0 and 1.
  - Do not take initial conditions into account.
- Generally, it is most useful to be able to predict the time it takes to reduce concentration variations by a certain amount.
- This is then done by scaling the largest deviation in mass fraction at time *t* by the largest deviation at time *t*=0.

$$U(t) = 1 - \frac{\Delta_{\max}(t)}{\Delta_{\max}(t=0)}$$

- E.g. for the example case:
  - At t=0s, Y_{max}=1 and $\langle Y \rangle$=0.0943 $\rightarrow \Delta_{max}(t=0)$ = 0.906.
  - At t=10s, Δ_{max}(10s) = 0.44 $\rightarrow$ U(10s) = 0.51.

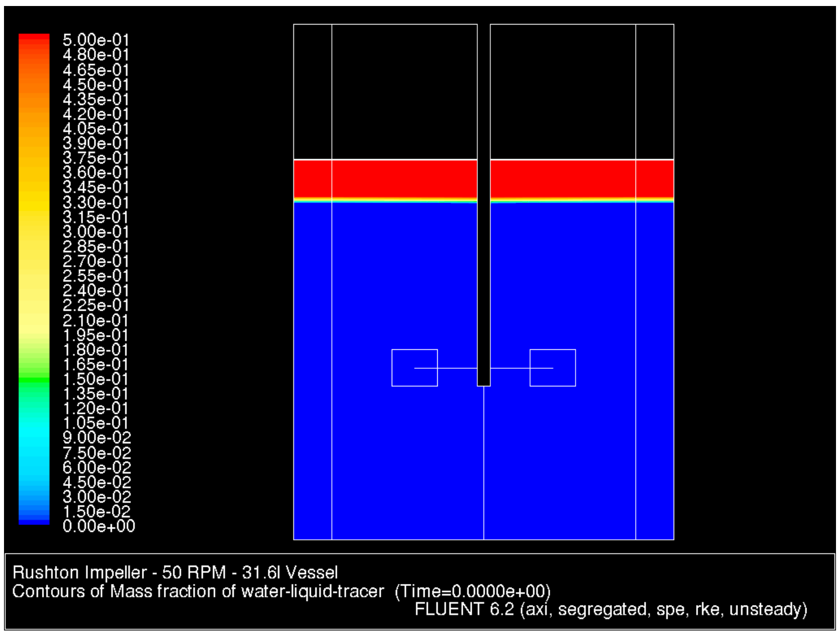
- Data are often correlated in terms of number of impeller revolutions, at t=10s and 50RPM, there were 10*RPM/60=8.33 impeller revolutions.



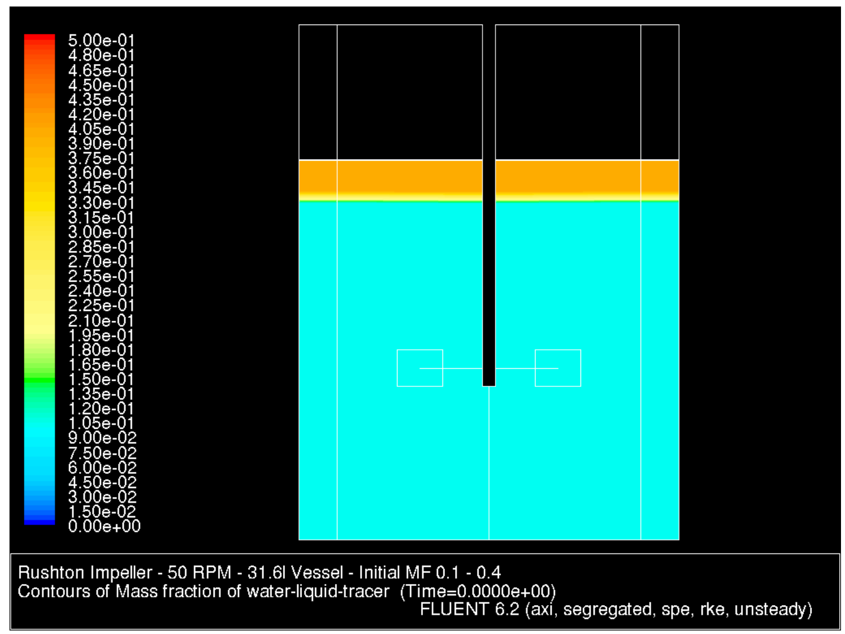
1.0
0.8
0.6
0.4
0.2
0.0
U
90% mixing time
99% mixing time
Blend time is then the time it takes to reduce the initial variation by a given percentage.
0
5
10
15
20
25
30
35
40
Impeller revolutions

Comparison between systems

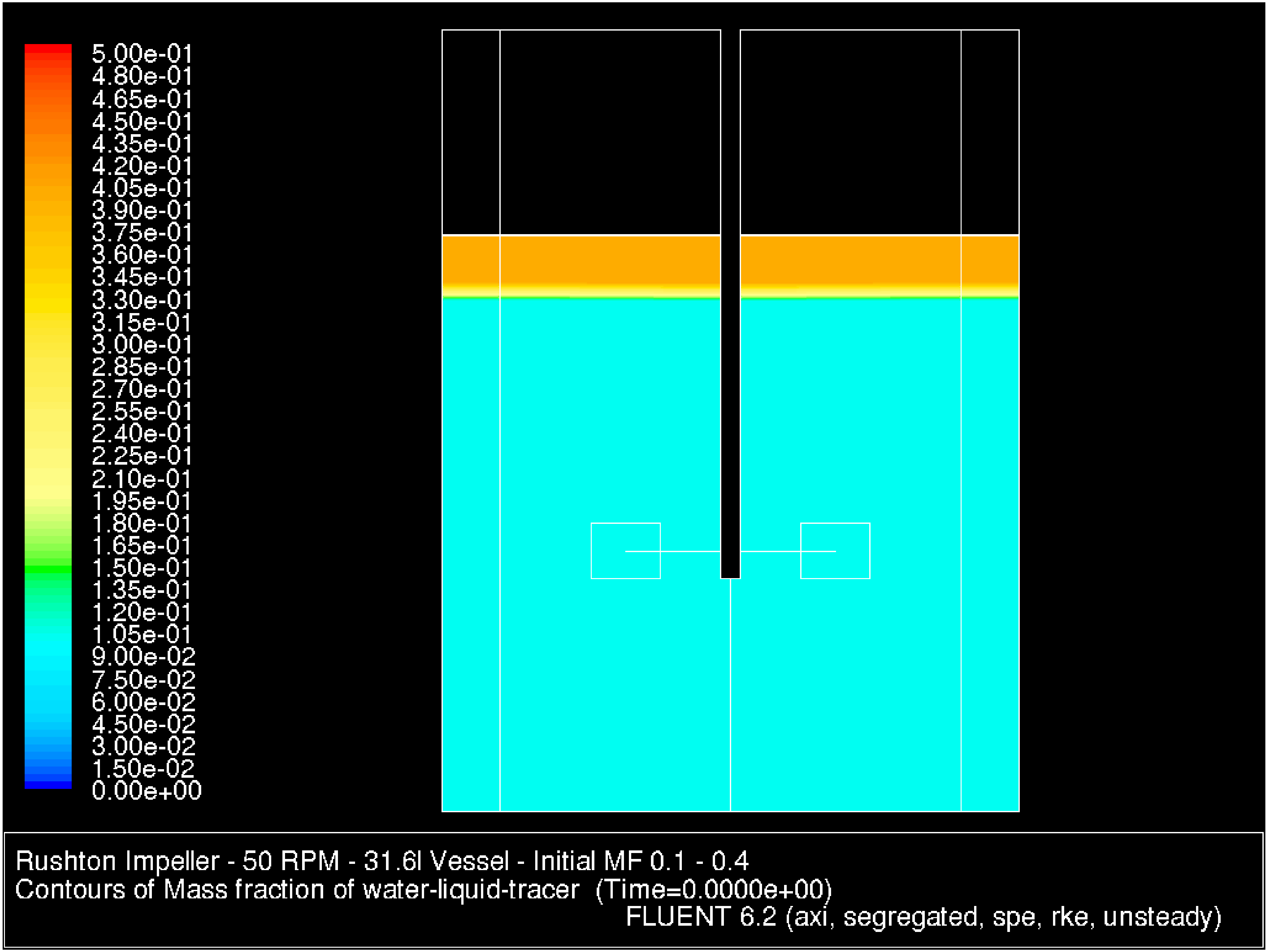
- Let's compare two systems with:
  - The same flow field.
  - The same spatial distribution of species.
  - But different initial mass fractions of species.

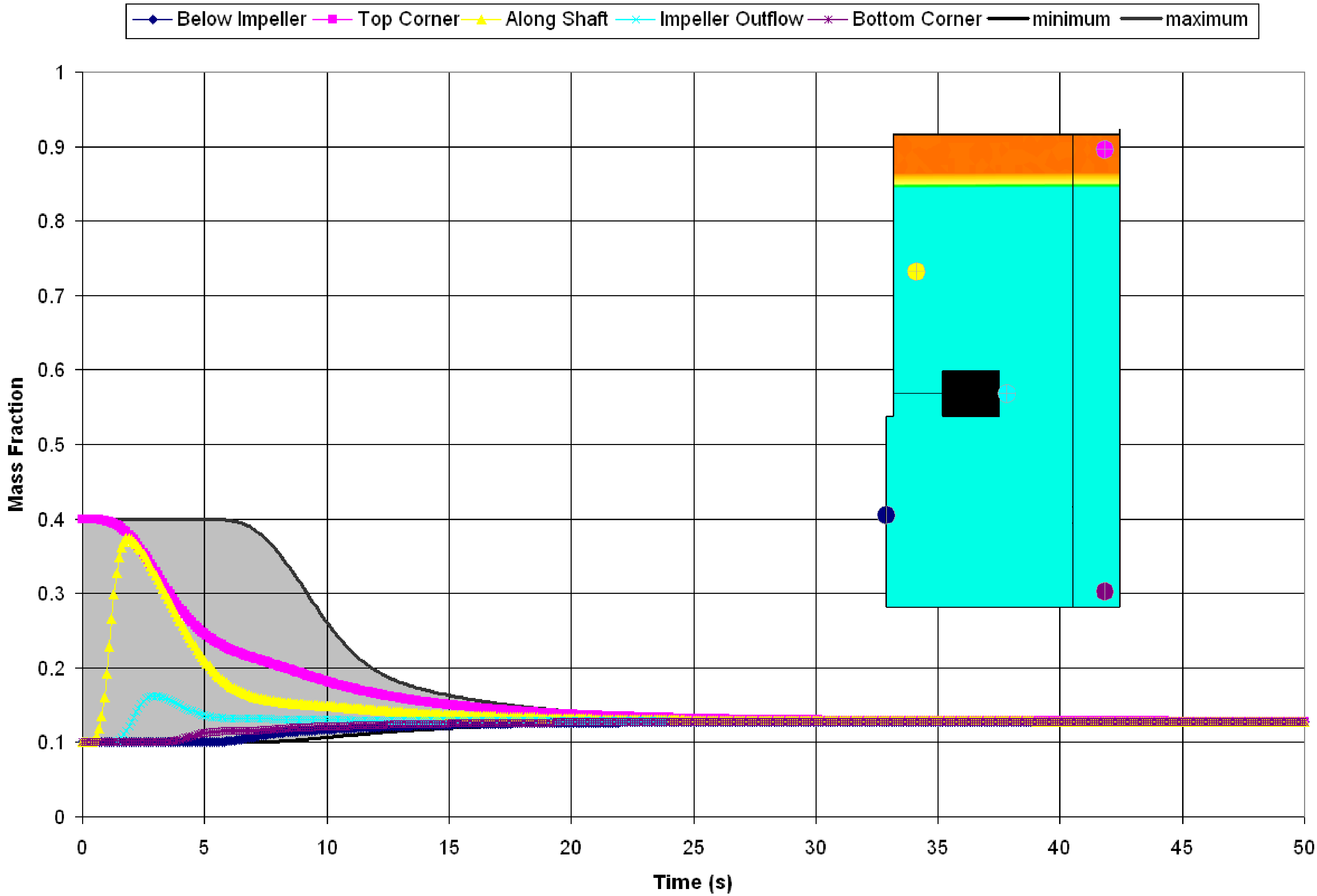


Layer with Y_{tracer}=1 on top of fluid with Y_{tracer} =0. <Y>=0.094.

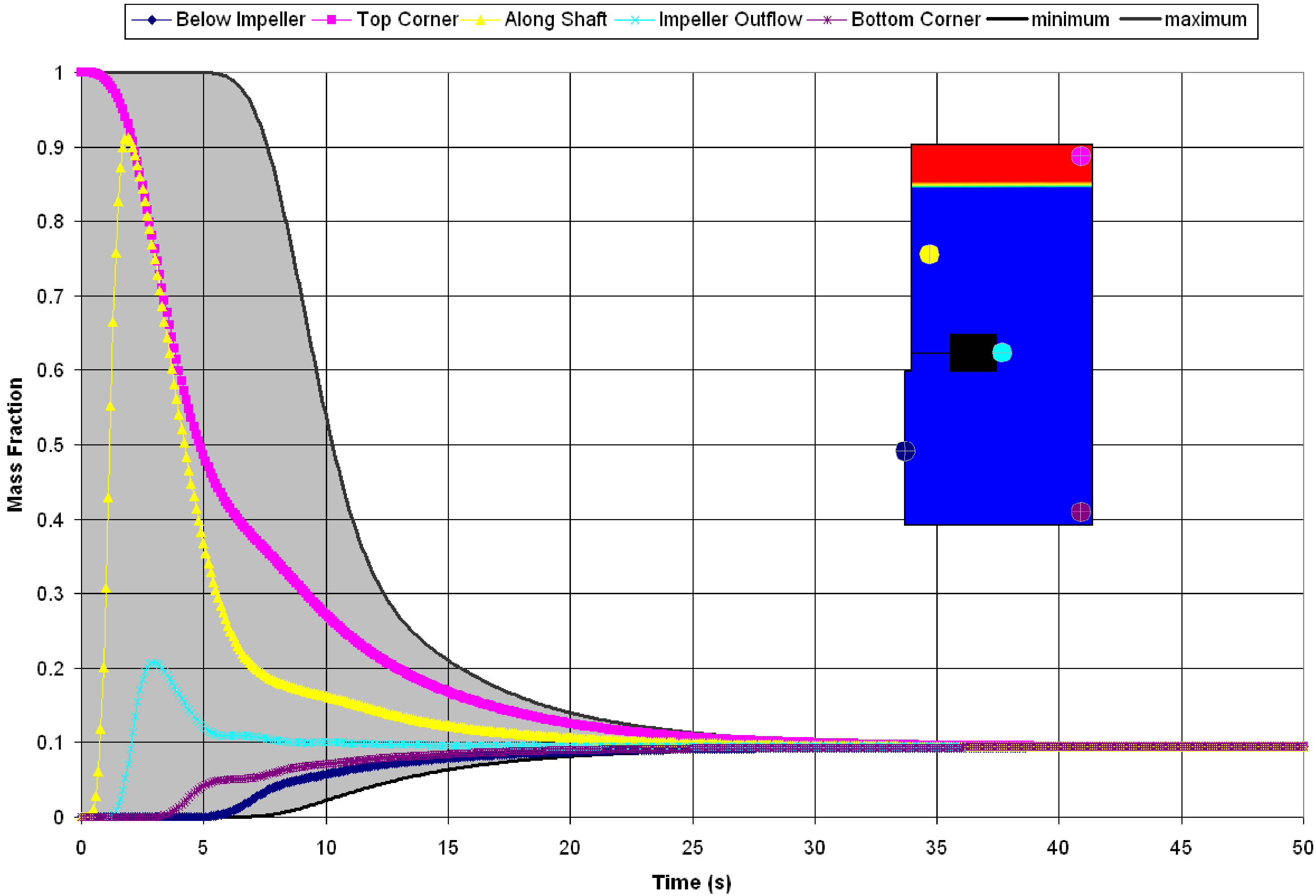


Layer with Y_{tracer}=0.4 on top of fluid with Y_{tracer} =0.1. <Y>=0.13.

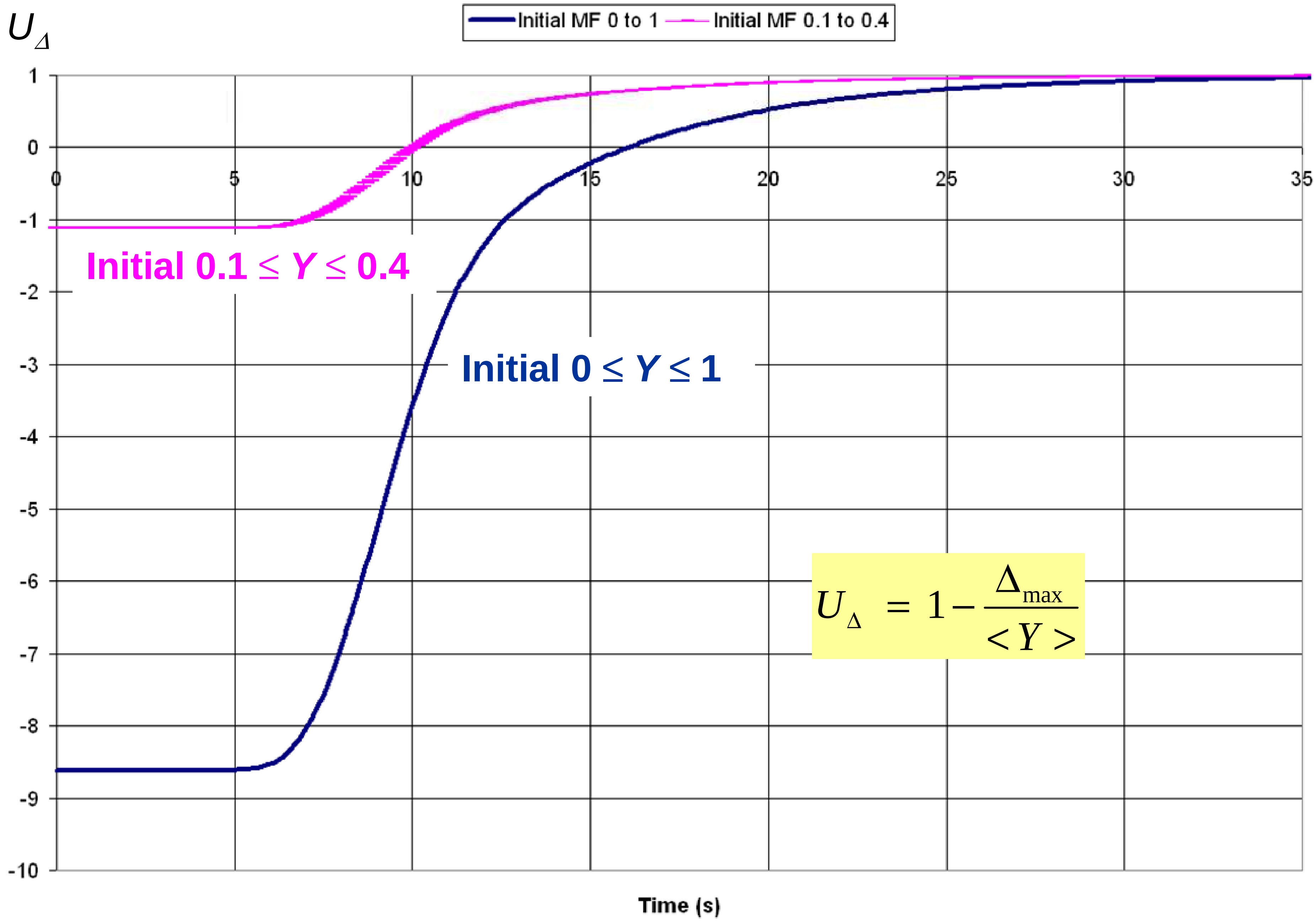

5.00e-01
4.80e-01
4.65e-01
4.50e-01
4.35e-01
4.20e-01
4.05e-01
3.90e-01
3.75e-01
3.60e-01
3.45e-01
3.30e-01
3.15e-01
3.00e-01
2.85e-01
2.70e-01
2.55e-01
2.40e-01
2.25e-01
2.10e-01
1.95e-01
1.80e-01
1.65e-01
1.50e-01
1.35e-01
1.20e-01
1.05e-01
9.00e-02
7.50e-02
6.00e-02
4.50e-02
3.00e-02
1.50e-02
0.00e+00
Rushton Impeller - 50 RPM - 31.6l Vessel - Initial MF 0.1 - 0.4
Contours of Mass fraction of water-liquid-tracer (Time=0.0000e+00)
FLUENT 6.2 (axi, segregated, spe, rke, unsteady)



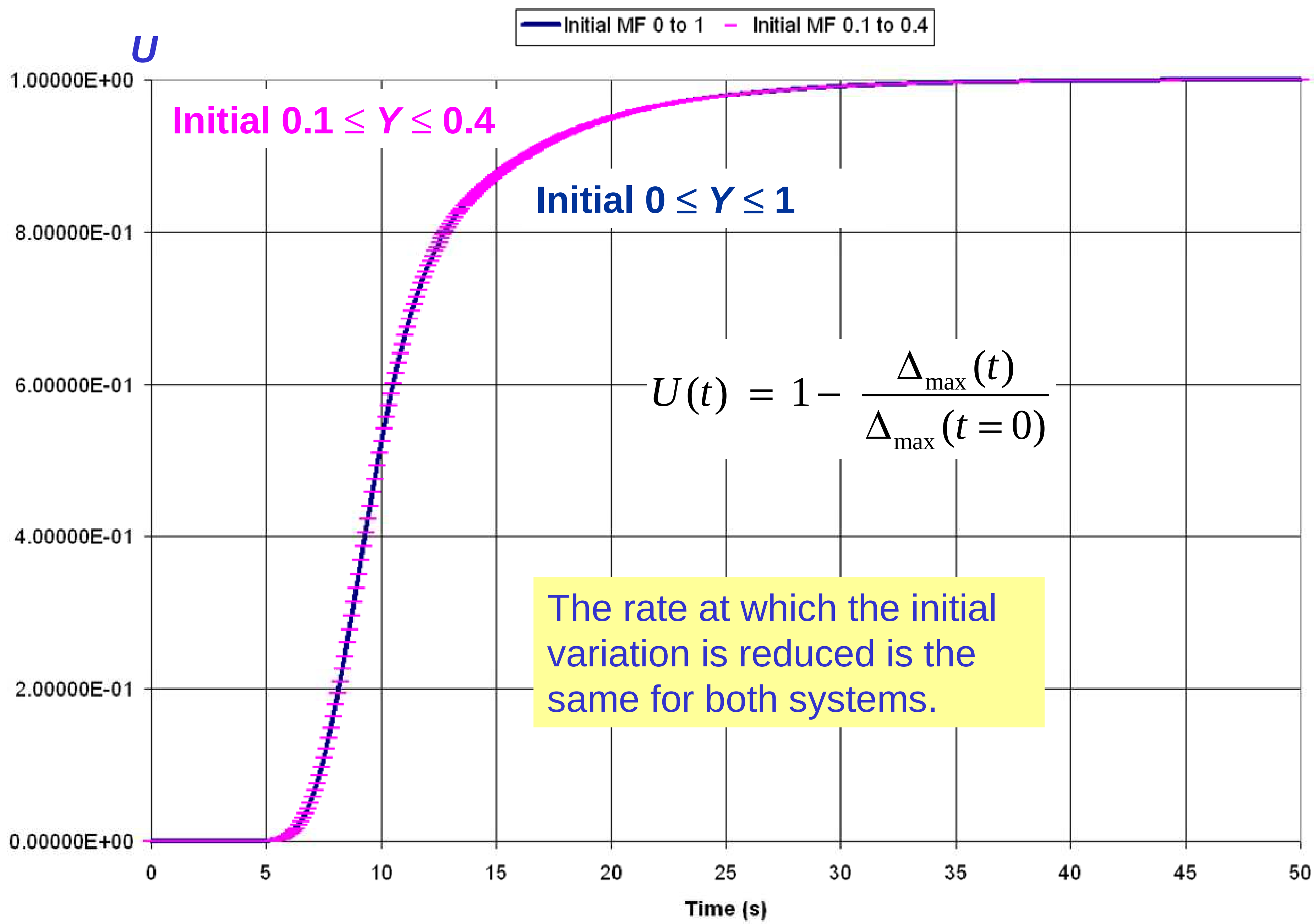
Below Impeller
Top Corner
Along Shaft
Impeller Outflow
Bottom Corner
minimum
maximum
Mass Fraction
Time (s)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
5
10
15
20
25
30
35
40
45
50



Below Impeller
Top Corner
Along Shaft
Impeller Outflow
Bottom Corner
minimum
maximum
Mass Fraction
Time (s)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
5
10
15
20
25
30
35
40
45
50



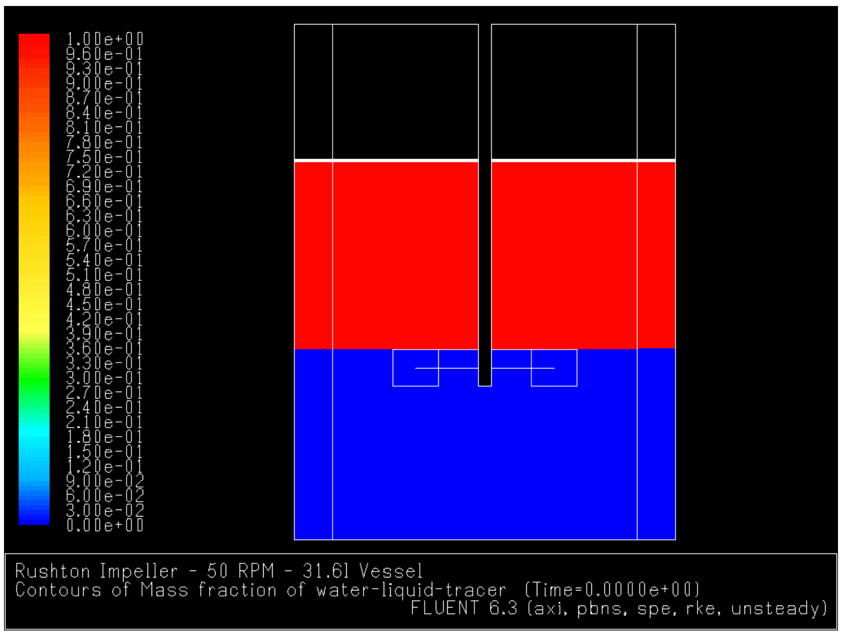
U_Δ
Initial MF 0 to 1
Initial MF 0.1 to 0.4
1
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
0
5
10
15
20
25
30
35
Initial $0.1 \le Y \le 0.4$
Initial $0 \le Y \le 1$
$U_\Delta = 1 - \frac{\Delta_{max}}{<Y>}$
Time (s)



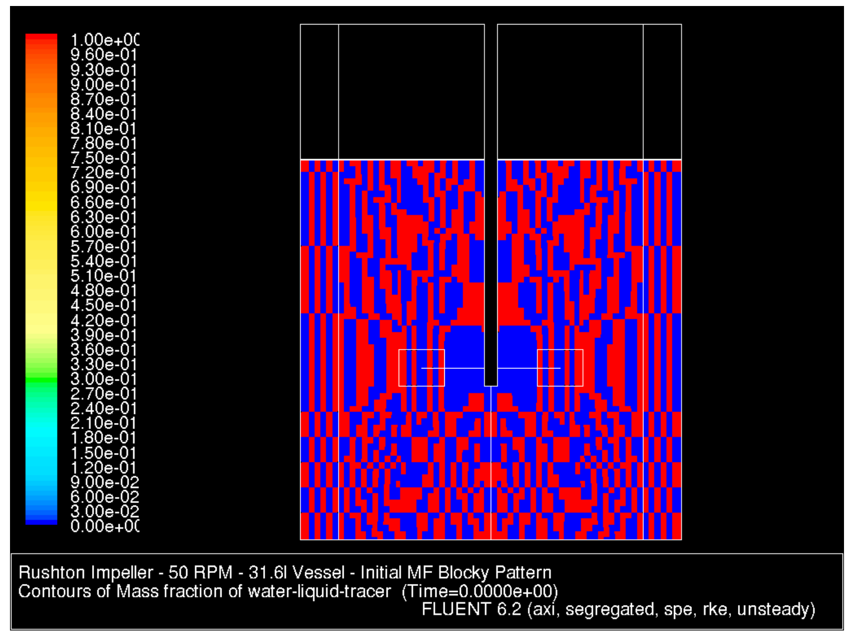
Initial MF 0 to 1
Initial MF 0.1 to 0.4
U
1.00000E+00
8.00000E-01
6.00000E-01
4.00000E-01
2.00000E-01
0.00000E+00
Initial 0.1 ≤ Y ≤ 0.4
Initial 0 ≤ Y ≤ 1
$U(t) = 1 - \frac{\Delta_{\max}(t)}{\Delta_{\max}(t=0)}$
The rate at which the initial variation is reduced is the same for both systems.
0
5
10
15
20
25
30
35
40
45
50
Time (s)

Compare two more systems

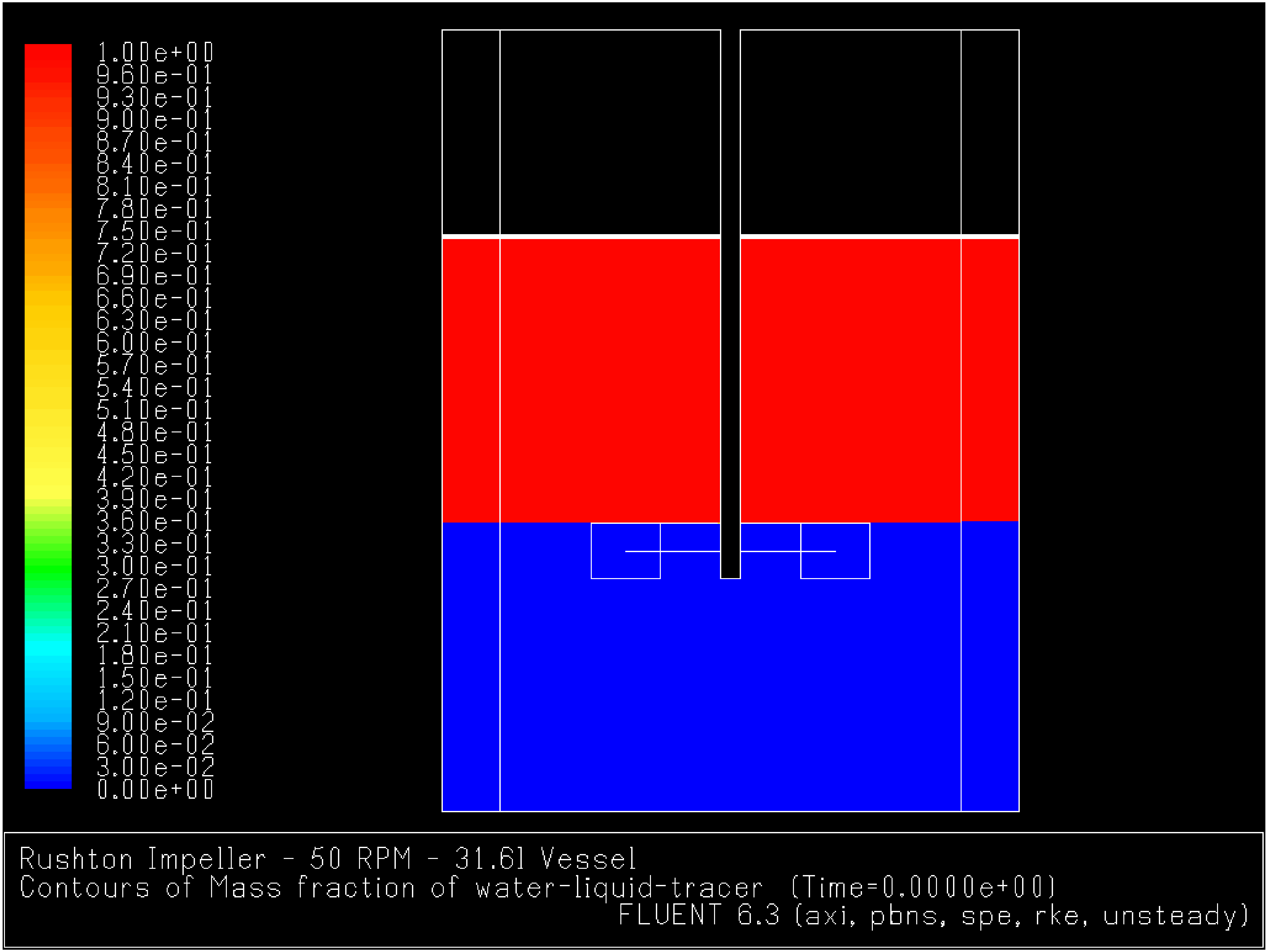
- Two systems with approximately the same average mass fraction of tracer <Y> ≅ 0.5.
- The initial distributions are very different: layered vs. blocky pattern.

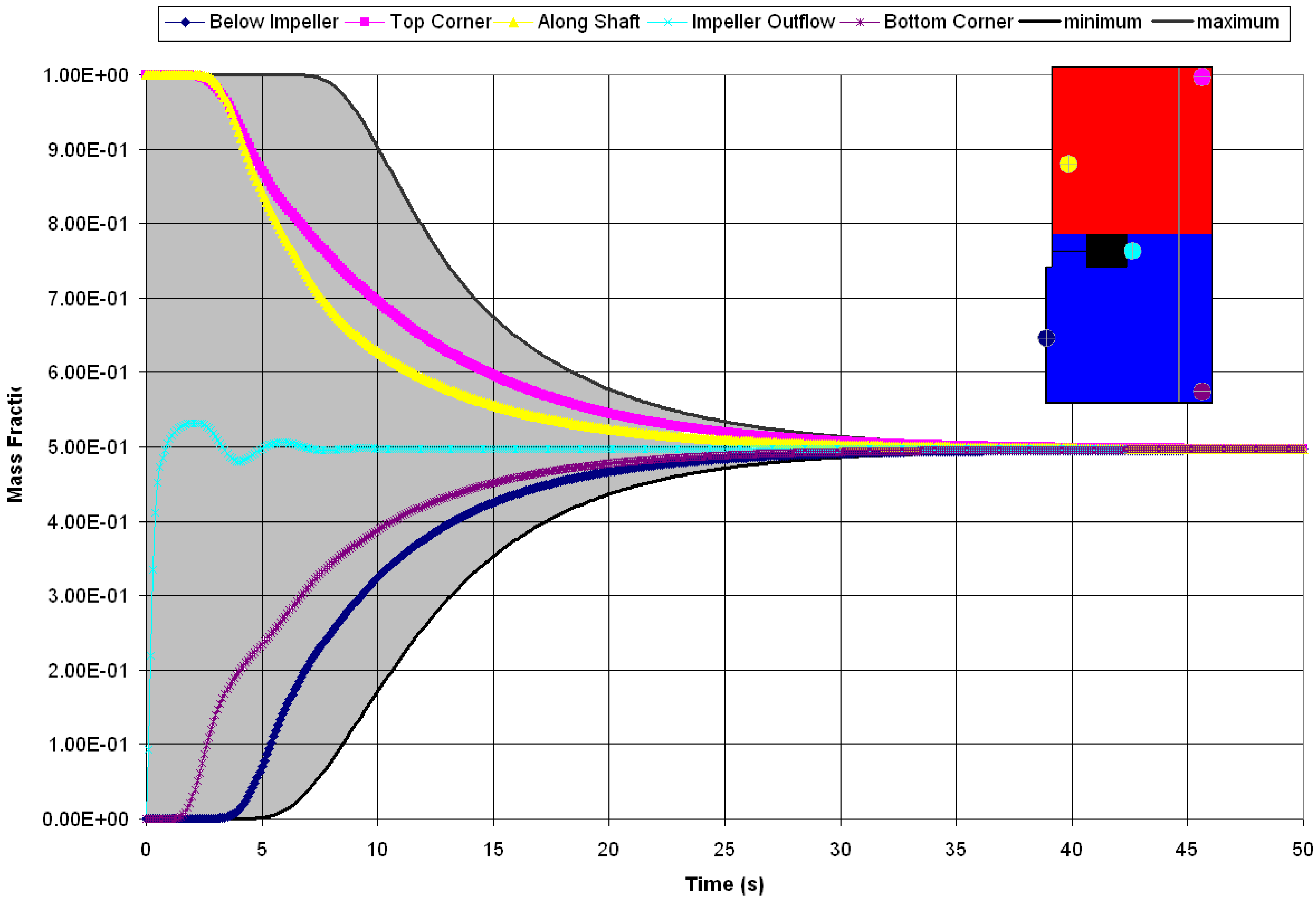


Layer with Y_{tracer}=1 on top of fluid with Y_{tracer} =0. <Y>=0.497.

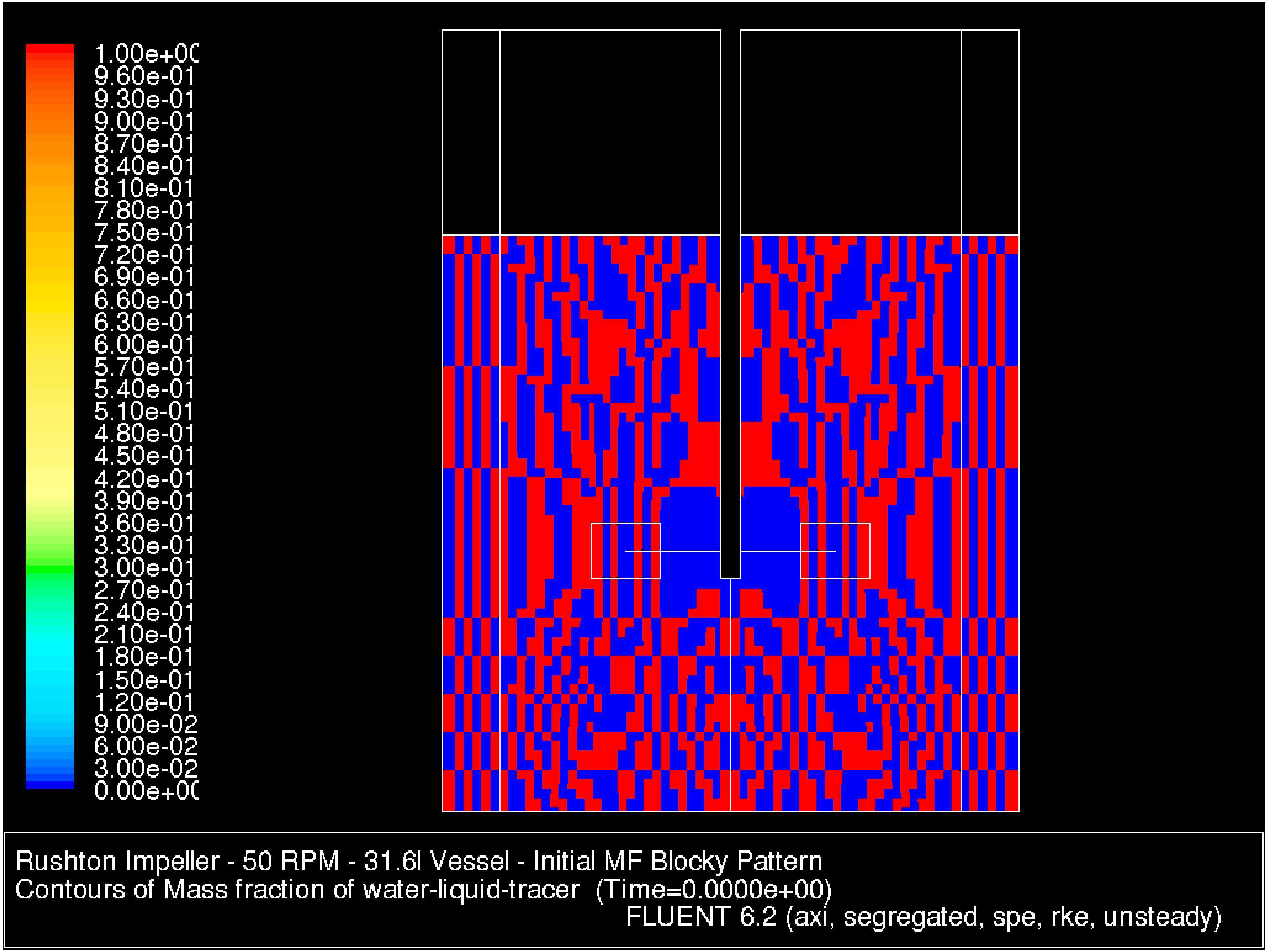


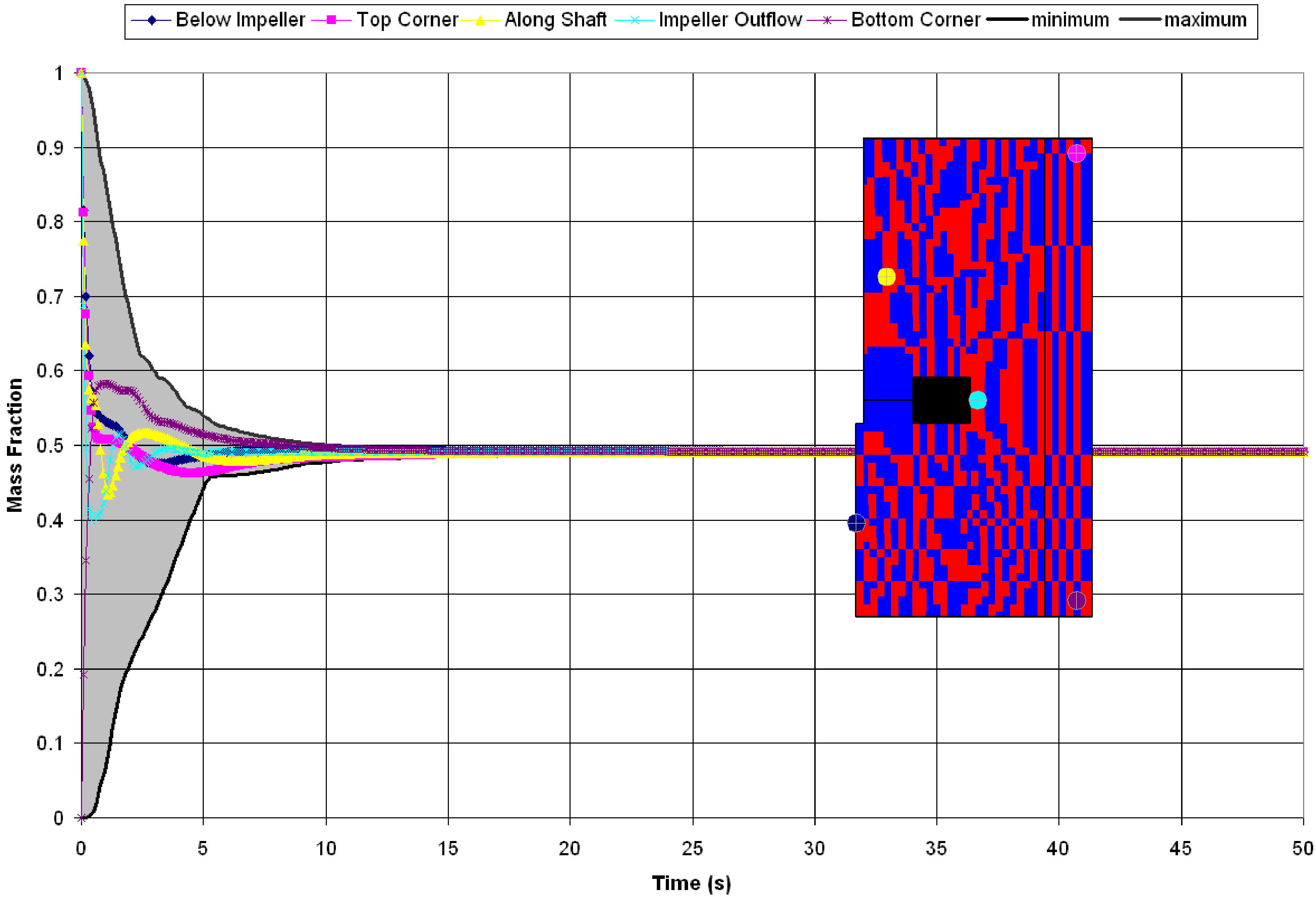
Blocky pattern of fluid with Y_{tracer}=0. and fluid with Y_{tracer} =1. <Y>=0.491.

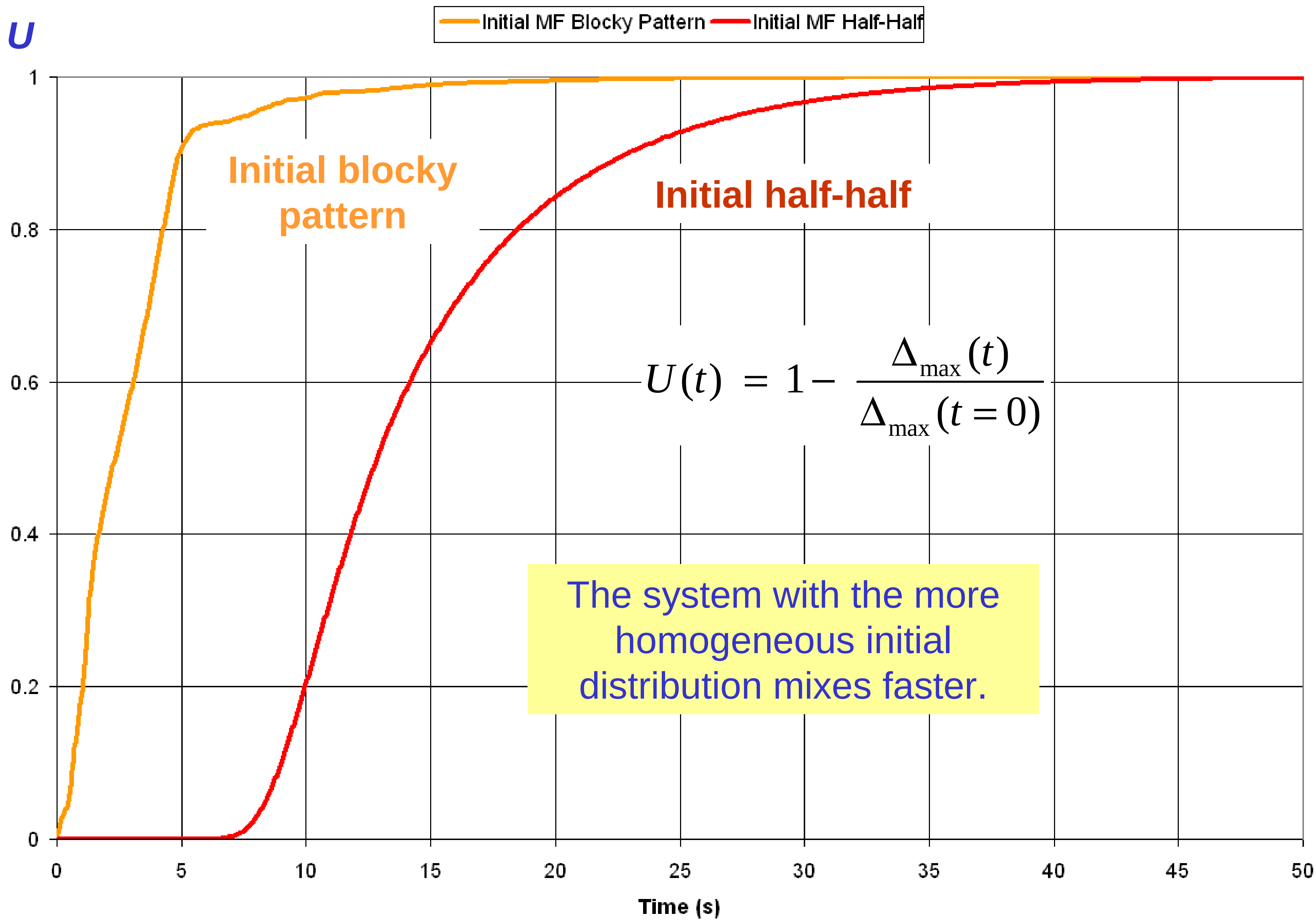

1.00e+00
9.60e-01
9.30e-01
9.00e-01
8.70e-01
8.40e-01
8.10e-01
7.80e-01
7.50e-01
7.20e-01
6.90e-01
6.60e-01
6.30e-01
6.00e-01
5.70e-01
5.40e-01
5.10e-01
4.80e-01
4.50e-01
4.20e-01
3.90e-01
3.60e-01
3.30e-01
3.00e-01
2.70e-01
2.40e-01
2.10e-01
1.80e-01
1.50e-01
1.20e-01
9.00e-02
6.00e-02
3.00e-02
0.00e+00
Rushton Impeller - 50 RPM - 31.6l Vessel
Contours of Mass fraction of water-liquid-tracer (Time=0.0000e+00)
FLUENT 6.3 (axi, pbns, spe, rke, unsteady)



Below Impeller
Top Corner
Along Shaft
Impeller Outflow
Bottom Corner
minimum
maximum
1.00E+00
9.00E-01
8.00E-01
7.00E-01
6.00E-01
5.00E-01
4.00E-01
3.00E-01
2.00E-01
1.00E-01
0.00E+00
Mass Fractic
0
5
10
15
20
25
30
35
40
45
50
Time (s)


1.00e+00
9.60e-01
9.30e-01
9.00e-01
8.70e-01
8.40e-01
8.10e-01
7.80e-01
7.50e-01
7.20e-01
6.90e-01
6.60e-01
6.30e-01
6.00e-01
5.70e-01
5.40e-01
5.10e-01
4.80e-01
4.50e-01
4.20e-01
3.90e-01
3.60e-01
3.30e-01
3.00e-01
2.70e-01
2.40e-01
2.10e-01
1.80e-01
1.50e-01
1.20e-01
9.00e-02
6.00e-02
3.00e-02
0.00e+00
Rushton Impeller - 50 RPM - 31.6l Vessel - Initial MF Blocky Pattern
Contours of Mass fraction of water-liquid-tracer (Time=0.0000e+00)
FLUENT 6.2 (axi, segregated, spe, rke, unsteady)


Below Impeller
Top Corner
Along Shaft
Impeller Outflow
Bottom Corner
minimum
maximum
Mass Fraction
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
5
10
15
20
25
30
35
40
45
50
Time (s)



U
Initial MF Blocky Pattern
Initial MF Half-Half
1
0.8
0.6
0.4
0.2
0
0 5 10 15 20 25 30 35 40 45 50
Time (s)
Initial blocky pattern
Initial half-half
$U(t) = 1 - \frac{\Delta_{max}(t)}{\Delta_{max}(t=0)}$
The system with the more homogeneous initial distribution mixes faster.

Compare all four systems

Initial Y	U
Layer (0 to 1)	20.3
Layer (0.1 to 0.4)	20.3
Blocky Pattern	10.7
Half-Half	26.1

- Table shows number of impeller revolutions it takes to achieve 99% uniformity for all four systems using the two main criteria:
  - *U* based on reduction in initial variation.
  - U_Δ based on variation from the average.
- Conclusion: systems with good initial distributions mix faster.
- General recommendation: use *U* (reduction in initial variation) to correlate results or to compare with literature blend time correlations.

Continuous systems

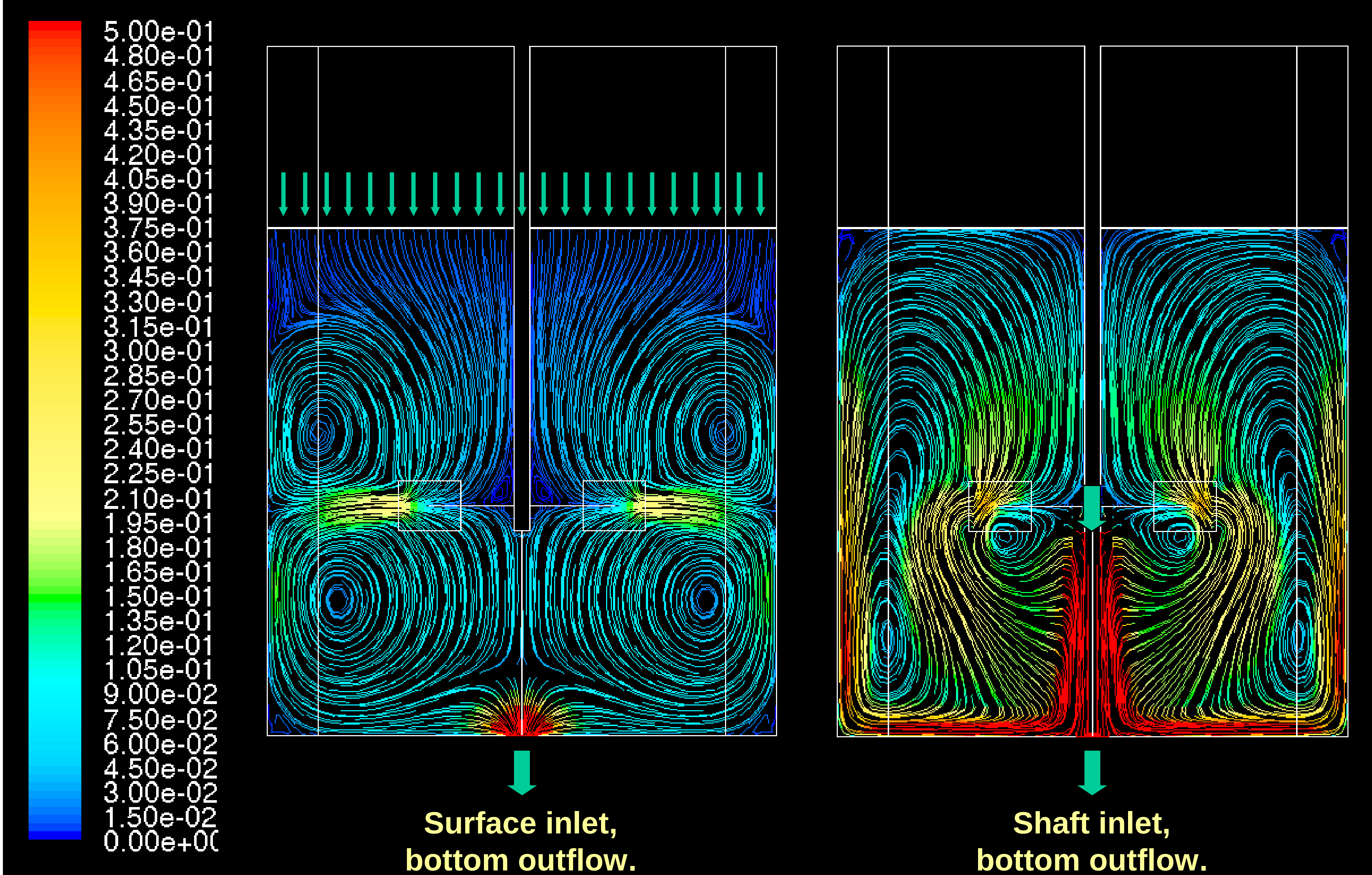
- Methods so far are for batch systems.
- Do these methods work for continuous systems?
  - Requires some modification.
  - Looking at mass fraction extremes does not work, because these may be fixed by the inlet mass fractions.
- Various approaches used:
  - Compare batch blend time with average residence time of the material (RT = liquid volume divided by volumetric flow rate). If batch blend time is much smaller than RT, assume there is no mixing problem.
  - Perform particle tracking simulation, similar to shown for static mixers in previous lectures. Analyze residence time distributions.
  - Perform tracer mixing calculation.

Tracer mixing calculation

- Calculate continuous, steady state flow field.
- Initial mass fraction of tracer is zero everywhere.
- Perform transient calculation for tracer mixing, with non-zero mass fraction tracer at inlet.
- Monitor:
  - Average mass fraction in domain $<Y>$.
  - Mass fraction at outlet Y_{out}.
  - Optional: monitor CoV.
- Definition of perfectly mixed system: $Y_{out} = <Y>$.
- Mixing time is then the time it takes for the ratio $Y_{out}/<Y>$ to be within a specified tolerance of 1.
- Mixing time can be expressed in number of residence times: t/RT.

Compare two systems

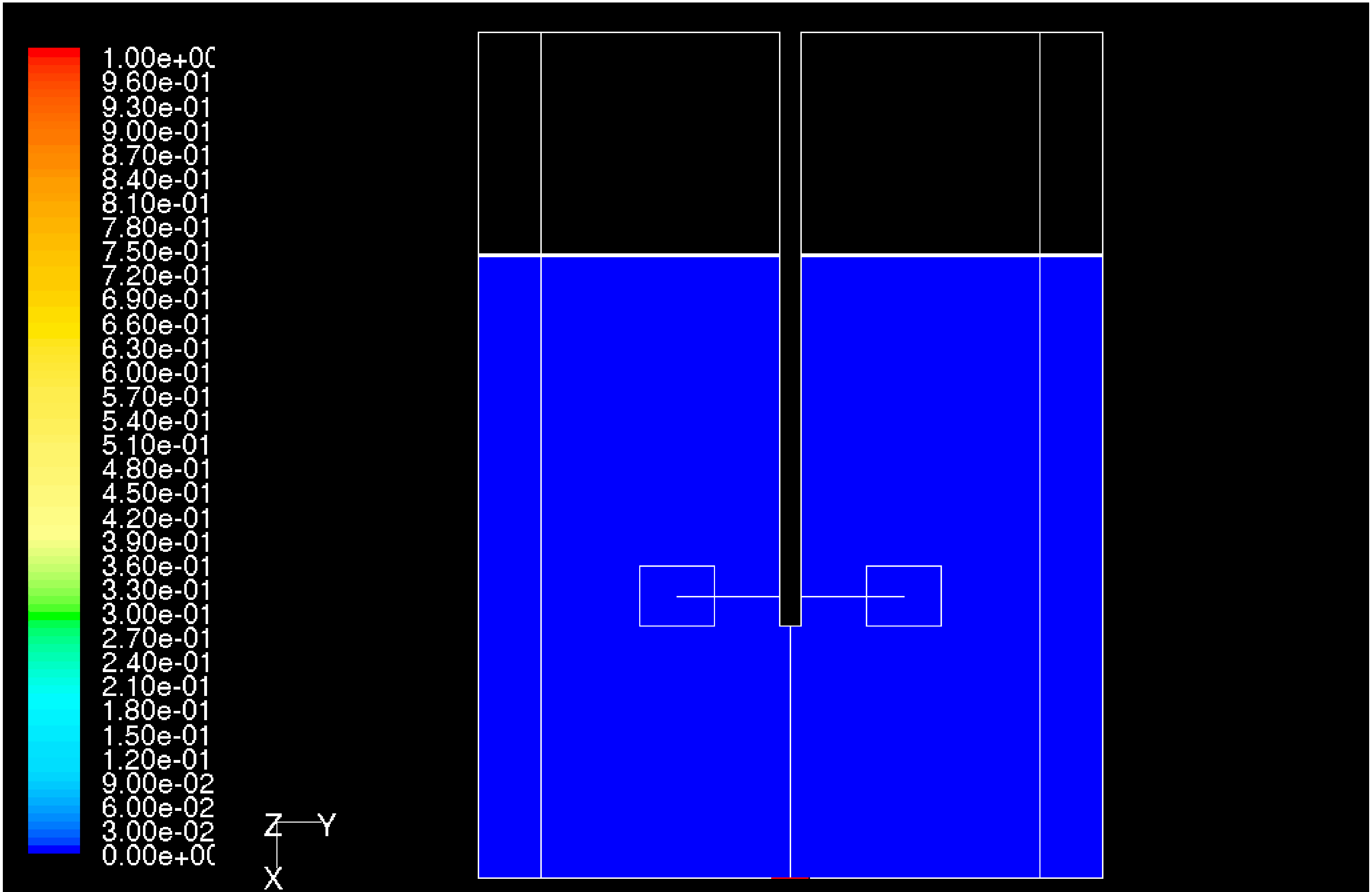
- Rushton turbine flow field.
- Continuous system with two different injections:
  - Low velocity feed (0.01 m/s) distributed across liquid surface.
    - Affects flow in upper part of the vessel only.
  - High speed jet feed (9.6 m/s) entering through bottom shaft.
    - Because of the large momentum contained in the jet, it alters the flow field significantly.
- Outflow at center of bottom.
- Average residence time RT=30s, equivalent to 25 impeller revolutions. The RT is similar to the batch blend time.



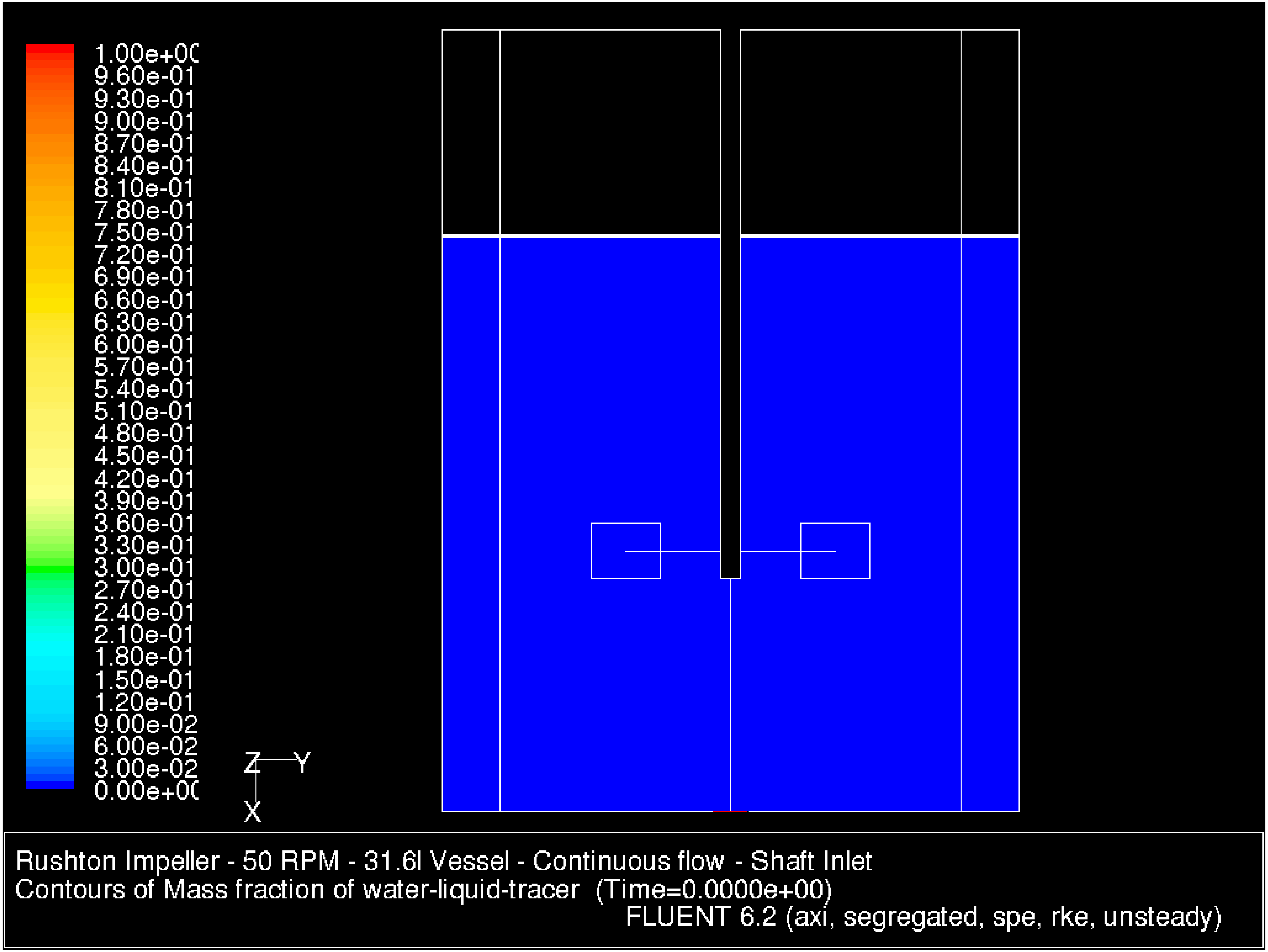
Rushton Impeller - 50 RPM - 31.6l Vessel - Continuous flow
Path Lines Colored by Velocity Magnitude (m/s)

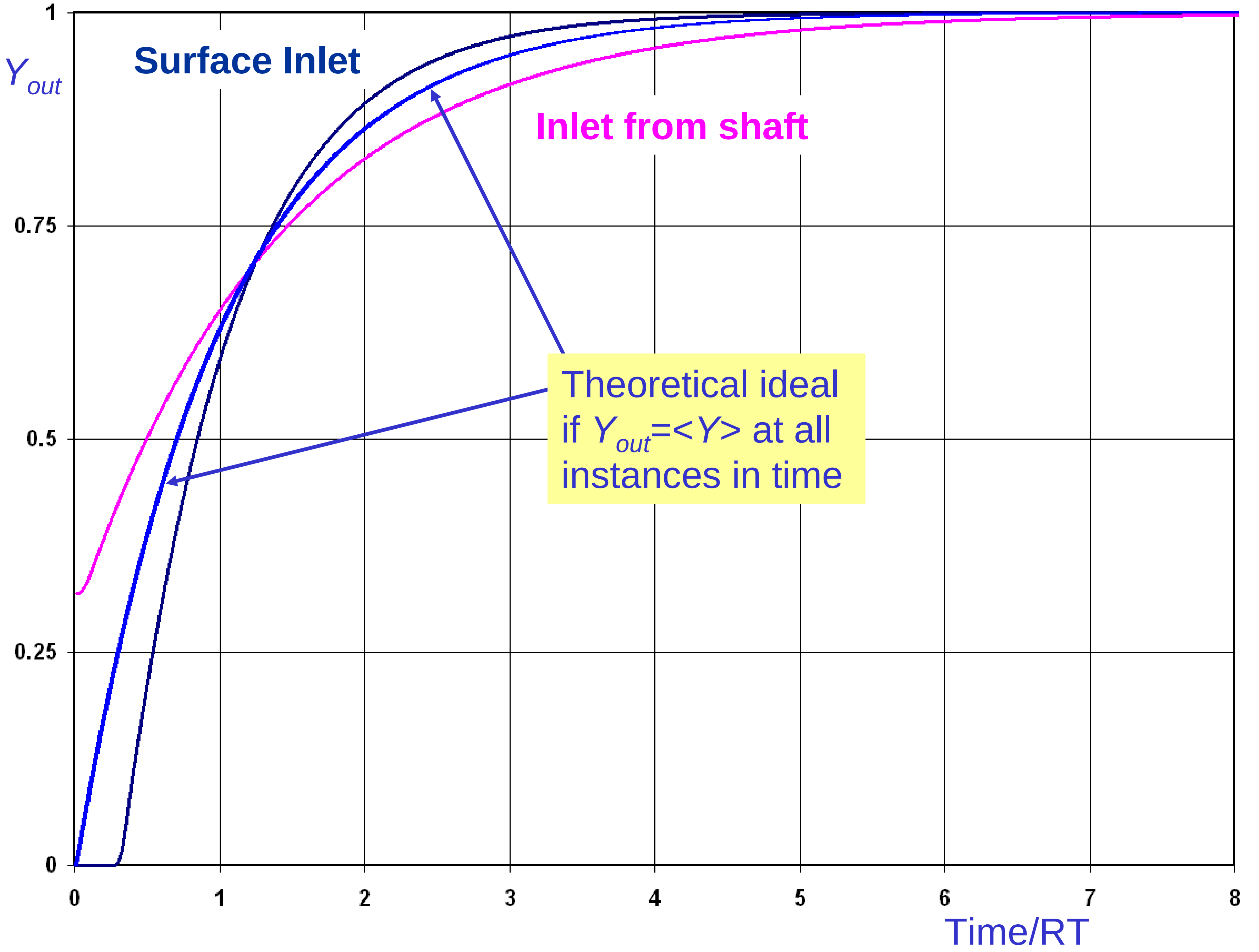
Average residence time RT=30s.

FLUENT 6.2 (axi, segregated, spe, rke)

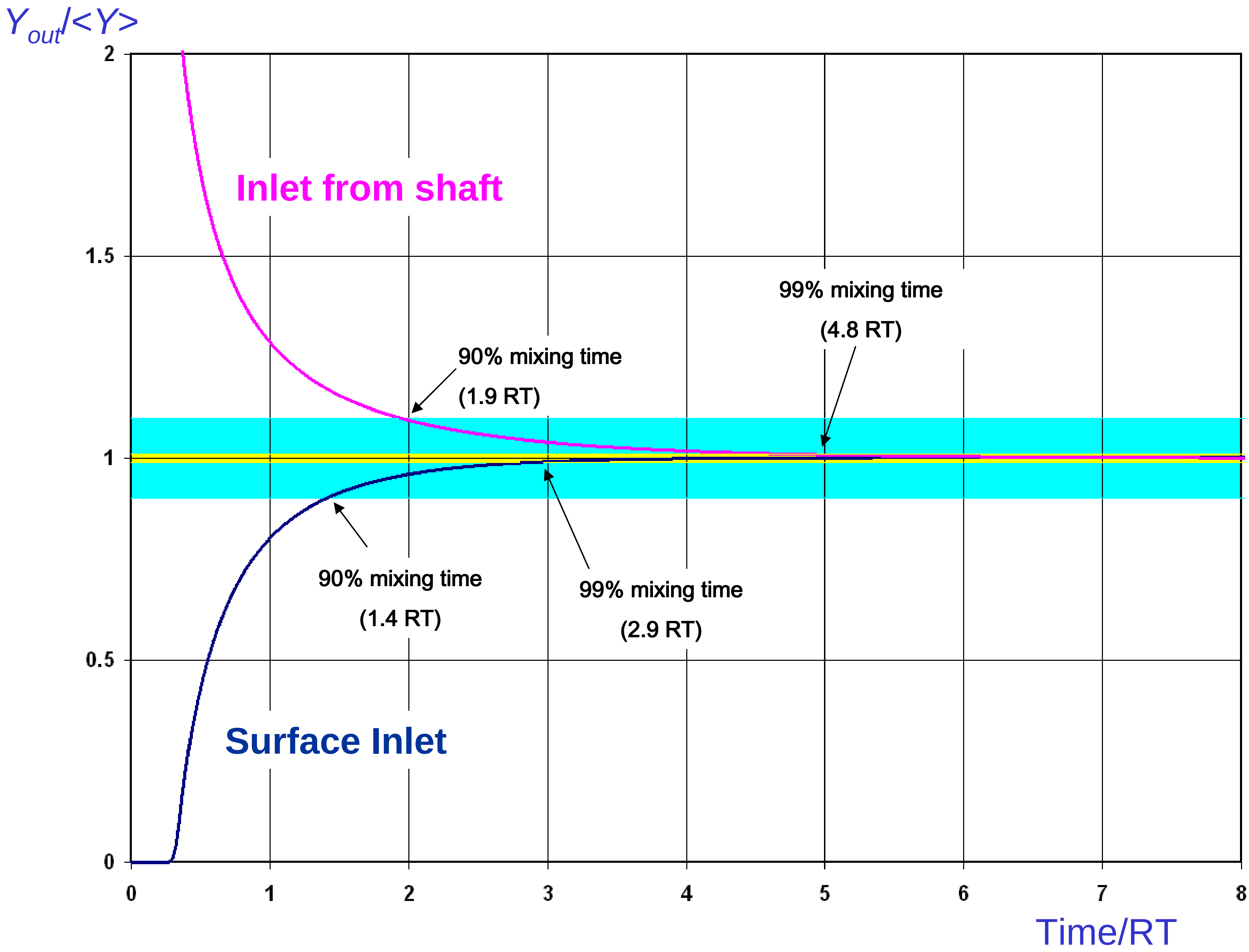


Rushton Impeller - 50 RPM - 31.6l Vessel - Continuous flow - Surface Inlet
Contours of Mass fraction of water-liquid-tracer (Time=0.0000e+00)
FLUENT 6.2 (axi, segregated, spe, rke, unsteady)


1.00e+00
9.60e-01
9.30e-01
9.00e-01
8.70e-01
8.40e-01
8.10e-01
7.80e-01
7.50e-01
7.20e-01
6.90e-01
6.60e-01
6.30e-01
6.00e-01
5.70e-01
5.40e-01
5.10e-01
4.80e-01
4.50e-01
4.20e-01
3.90e-01
3.60e-01
3.30e-01
3.00e-01
2.70e-01
2.40e-01
2.10e-01
1.80e-01
1.50e-01
1.20e-01
9.00e-02
6.00e-02
3.00e-02
0.00e+00
Z
Y
X
Rushton Impeller - 50 RPM - 31.6l Vessel - Continuous flow - Shaft Inlet
Contours of Mass fraction of water-liquid-tracer (Time=0.0000e+00)
FLUENT 6.2 (axi, segregated, spe, rke, unsteady)



1
0.75
0.5
0.25
0
Y_{out}
Surface Inlet
Inlet from shaft
Theoretical ideal
if Y_{out}=<Y> at all
instances in time
0
1
2
3
4
5
6
7
8
Time/RT



Yout/<Y>
2
1.5
1
0.5
0
Inlet from shaft
99% mixing time
(4.8 RT)
90% mixing time
(1.9 RT)
90% mixing time
(1.4 RT)
99% mixing time
(2.9 RT)
Surface Inlet
0
1
2
3
4
5
6
7
8
Time/RT

Comments

- The main assumption behind this approach is that the system will eventually reach a steady state where $Y_{out}=\langle Y\rangle$.
  - Not all industrial systems may have a steady state operating condition which, in general, is an undesirable situation that would need to be addressed.
- CoV can still be used to compare uniformity of different systems under steady state operating conditions with multiple species.