CHEG 615 - Special Topics in Mixing

Lecture 7
Liquid-Liquid Mixing

Uses for Liquid-Liquid Mixing

- **Uses**
  - **Contacting for Mass Transfer**
  - **Single Mixer-Settler**
  - **Staged Columns**
- **Products**
  - **Stable Emulsions**
  - **Food and Cosmetic Products**

Multiphase Systems

- **Solid-Liquid**
  - **Fixed Surface Area**
  - **Gravity plays moderate role**
- **Gas-Liquid**
  - **Variable Surface Area**
  - **Gravity plays strong role**
- **Liquid-Liquid**
  - **Variable Surface Area**
  - **Gravity plays weak role**
- **Mass Transfer**
  - **Similarity in mass transfer correlations**

Multiphase Flow Regimes

- **Back-mixed Continuous (Liquid) Phase**
  - **Solid-Liquid, Liquid-Liquid, Gas-Liquid**
- **Back-mixed Dispersed Phase**
  - **Solid-Liquid, Liquid-Liquid**
- **Plug Flow Dispersed Phase**
  - **Gas-Liquid**
- **Challenges**
  - **What is the interfacial surface area?**
  - **What are the film coefficients?**

$$J = k_L a(C^* - C)$$

- **Equilibrium Stages often used with Liquid-Liquid Systems**
  - **Presumes mass transfer is fast and complete within residence time**

Liquid-Liquid Drop Breakup

- **Key Principle - Drops have a characteristic strength that depends on drop size**
  - **characterized by**

$$\frac{\sigma}{d} = \left[\frac{N \cdot m^{-1}}{m}\right] = \left[\frac{N}{m^2}\right] = \left[\frac{force}{area}\right] = [stress]$$

$$where\ \sigma = interfacial\ tension \quad and \quad d = drop\ diameter$$

- **In a given fluid stress field, drops will break until the characteristic strength can no longer be overcome**
  - **drops break down to a maximum size beyond which they will no longer break**
  - **not all drops will be that maximum size**
  - **there will be a drop size distribution**

Mass Transfer in Liquid-Liquid Systems

- **Mass Transfer Rate, J**

$$J = k_L a(C^* - C)$$

- **Average Drop Sizes**
  - **Various averages possible which differ bases on drop size distribution**
  - **Sauter mean diameter, d_{32} - average of surface area per unit volume of all drops**

$$d_{32} = \sum_i \frac{d_i^3}{d_i^2}$$

- **Need to estimate interfacial area, a**

$$a = \frac{6\phi}{d_{32}} \quad \textit{where } \phi = \textit{total volume of dispersed phase}$$

Drop Break-up in Low Viscosity Turbulent Flow

- **Drop Weber Number $(We)_d$**
  - **Break-up occurs above a critical $(We)_d$**

$$(We)_d = \frac{\tau_c}{(\sigma/d)} = \frac{[breakup \;\; stress]}{[stabilizing \;\; stress]}$$

- **Break-up Stress in Turbulent Flow - Reynolds Stress**

$$\tau_c \propto \rho_c (u'v') \approx \rho_c (u')^2$$

- **Assume drops much larger than Kolmogoroff length scale so affected by eddies in the inertial sub-range**

$$l_E \approx \frac{(u')^3}{\varepsilon} \quad \Rightarrow \quad u' \approx (\varepsilon \, l_E)^{1/3}$$

$$Assume \quad d_{\max} = l_E$$

Drop Break-up in Low Viscosity Turbulent Flow

- **At drop break-up, break-up stresses balance stabilizing stresses**

$$(We)_d = \frac{\tau_c}{(\sigma/d_{\max})} = \frac{\rho_c (\varepsilon d_{\max})^{2/3}}{(\sigma/d_{\max})} = const$$

$$\rho_c (\varepsilon d_{\max})^{2/3} \propto (\sigma/d_{\max})$$

$$d_{\max} \propto \left(\frac{\sigma}{\rho_c}\right)^{0.6} \varepsilon^{-0.4}$$

- **ε - local power per volume not average**
  - **In stirred tanks, use power per impeller swept volume**

$$Assume \quad \varepsilon \propto \frac{P}{\rho V_{imp}} = \frac{P_o \rho N^3 D^5}{\rho \frac{\pi D^2}{4} D_w} = \frac{P_o N^3 D^5}{\frac{\pi D^2}{4} \alpha D} = \frac{P_o}{\frac{\pi \alpha}{4}} N^3 D^2 \quad \Rightarrow \quad \varepsilon \propto N^3 D^2$$

- **Combine to give**

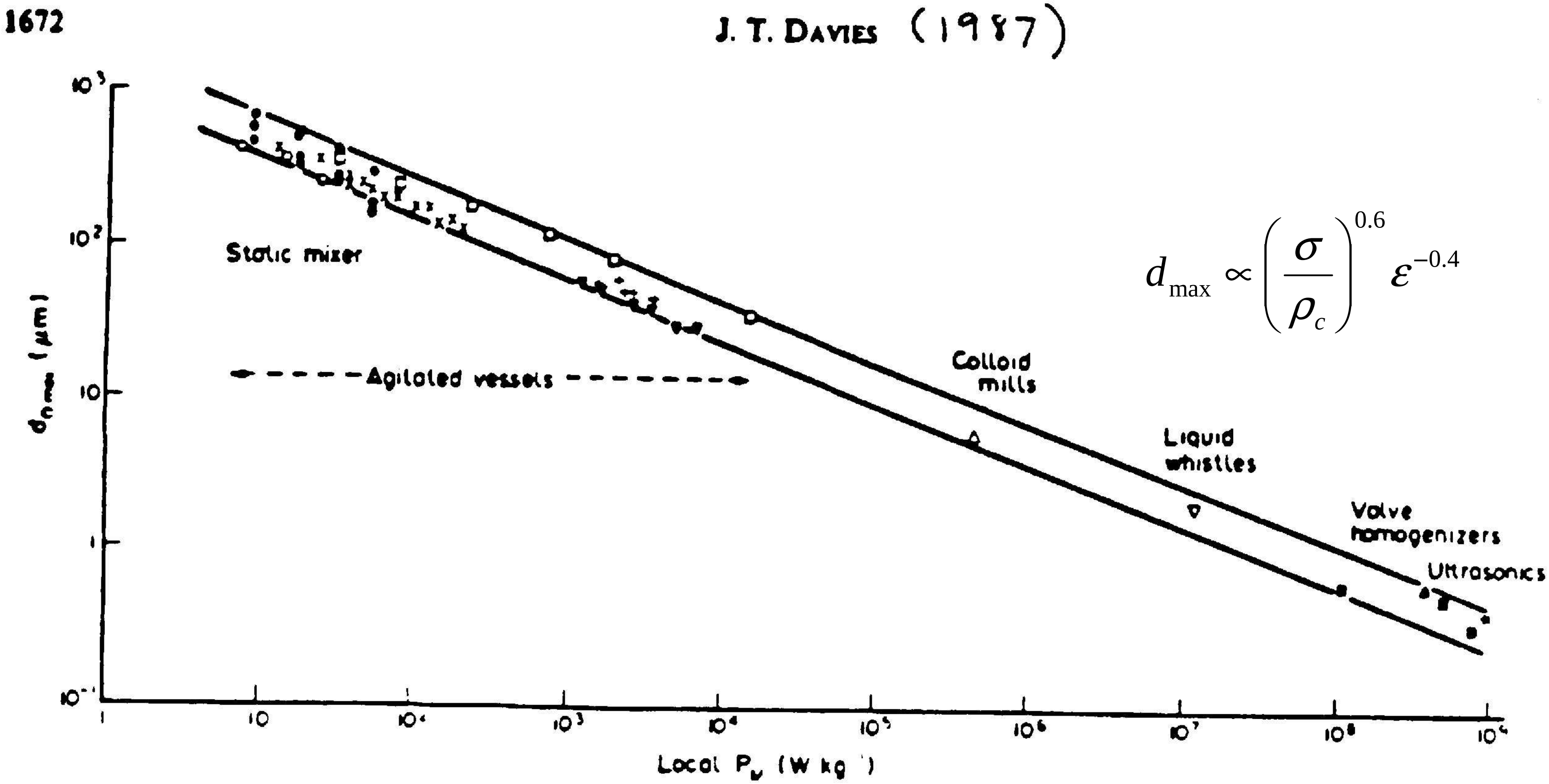
$$d_{\max} \propto \left(\frac{\sigma}{\rho_c}\right)^{0.6} (N^3 D^2)^{-0.4}$$

$$\frac{d_{\max}}{D} \propto \left(\frac{\rho_c N^2 D^3}{\sigma}\right)^{-0.6} \quad \Rightarrow \quad \frac{d_{\max}}{D} \propto (We)_D^{-0.6}$$

- **Studies show $d_{max} \propto d_{32}$**

$$d_{\max} = 1.7 d_{32} \quad (Davies\ correlation)$$

D_{max} vs. Power per Mass



$$d_{\max} \propto \left(\frac{\sigma}{\rho_c}\right)^{0.6} \varepsilon^{-0.4}$$

- **Stirred Tanks may not be best for liquid drop dispersion**
  - **wide distribution of eddy sizes and energy dissipation rates**
  - **bypassing**

Equilibrium Drop Size Distribution
Low to Moderate Dispersed Phase Viscosity

- **Sauter Mean Diameter including viscosity effects (Calabrese, Wang & Bryner, 1986)**
  - **349 data sets (Rushton turbine)**

$$\frac{d_{32}}{D} = 0.054 We^{-3/5}\left[1 + 4.42 Vi\left(\frac{d_{32}}{D}\right)^{1/3}\right]^{3/5}$$

$$where\ Vi = \frac{\mu_d ND}{\sigma}\left(\frac{\rho_c}{\rho_d}\right)^{1/2}$$

- **Cumulative Volume Frequency (Wang & Calabrese, 1986)**
  - **146 data sets**

$$F_v\left(\frac{D}{d_{32}}\right) = 0.5\left[1 + erf\left(\frac{D/d_{32} - 1.07}{0.24\sqrt{2}}\right)\right]$$

Minimum Dispersion Speed

- N_{JD} - minimum impeller speed to disperse liquid droplets
- N_{JD} Correlation for 6-blade disk impeller (Penney, et al., 1999)

$$S = \left[\left(\frac{N_{JD}^2 D}{g}\right)\left(\frac{D^2 \rho_c g}{\sigma}\right)^{1/2}\right]^{1/2} \left(\frac{\rho_h}{\rho_h - \rho_l}\right)^{1/4}$$

where

$$S = f\left(\frac{H}{Z}, \frac{C}{T}, \frac{D}{T}\right)$$

$$H = \text{height of light phase in static condition}, m$$

$$Z = \text{total liquid height}, m$$

$$\rho_h = \text{density of heavy phase}, kg/m^3$$

$$\rho_l = \text{density of light phase}, kg/m^3$$

$$\left(\frac{N_{JD}^2 D}{g}\right) = \text{Froude number } (Fr) = \frac{[\text{inertial forces}]}{[\text{gravitational forces}]}$$

$$\left(\frac{D^2 \rho_c g}{\sigma}\right) = \text{Goucher number } (Go) = \frac{[\text{gravitational forces}]}{[\text{interfacial forces}]}$$

Time to Reach Equilibrium Drop Size

- **Time to achieve equilibrium drop size**
  - **long times**
    - **5-20 minutes at lab-scale**
    - **hours at plant-scale**
- **Two parallel process concept (Baldyga & Bourne)**
  - **Breakup - fast process**
  - **Coalescence - slow process**
- **Rate of Drop Break-up**
  - **probability of drop going through impeller zone**
  - **circulation time**

$$t_Q = \frac{V}{Q}$$

  - **tank volume important**

Coalescence - Important Factors

- **Flow field and collision rate**
- **Volume fraction of the dispersed phase**
- **Viscosity of both phases**
- **Condition, age, viscosity and mobility of drop interfaces**
- **Presence of particulates, surfactants or suspending agents**
- **Coalescence can occur at many locations which complicates interpretation**
  - **impeller blades**
  - **baffles**
  - **liquid surface**

Population Balance Equation

$$\frac{\partial}{\partial t}[N(t)A(v,t)] = \int_v^{v_{max}} \beta(v',v)\nu(v')g(v')N(t)A(v',t)\,dv'$$
$$- g(v)N(t)A(v,t)$$
$$+ \int_0^{v/2} \lambda(v-v',v')h(v-v',v')N(t)A(v-v',t)N(t)A(v')\,dv'$$
$$- N(t)A(v,t)\int_0^{v_{max}-v} \lambda(v,v')h(v,v')N(t)A(v',t)\,dv'$$

$N(t) =$ total number of droplets in the vessel at time t

$A(v,t) =$ number probability density for drops of volume v at time t

$\beta(v',v) =$ brekage kernel; the number probability density of daughter drops of volume v formed by the breakup of a parent drop of volume v'

$\nu(v') =$ mean number of daughter drops resulting from breakage of a parent drop of volume v'

$g(v') =$ breakage frequency of a drop of volume v'

$\lambda(v,v') =$ collision efficiency of drops of volume v with drops of volume v'

$h(v,v') =$ collision frequency between drops of volume v and drops of volume v'

Separator (Decanter) Design

- **Liquid-liquid Flow into a Cylindrical Vessel**
- **Quiet Zone where interface forms**
- **Coalescence Layer or Band**
- **Assume drops in both phases**
- **Settle to coalesce and form interface**
- **Avoid interfering with settling process**
  - **Gravity separation**
  - **Settling rate given by Stoke's Law**

$$V_s = d^2\left(\frac{\rho_c}{\rho_d}\right)\frac{18g}{\mu_c}$$

- **Design for a certain cut size**
  - **for instance, get all drops above 125 microns**
- **Keep (through flow)/(velocity) equal to cut size settling rate**