Laminar Flow in Static Mixers with Helical Elements

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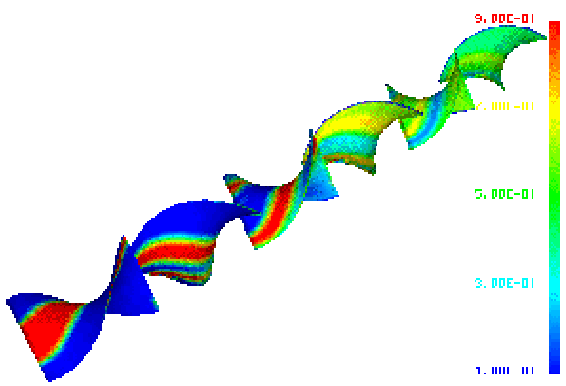

The flow pattern, pressure drop and the mixing characteristics of Kenics™ static mixers are investigated by means of computer simulations. The static mixer consists of a series of alternating left and right hand helical elements.

The simulations gave new insights in the flow pattern in the helical mixing elements. The pressure drop predictions compare favorably with literature data. Mixing in the elements occurs through a combination of flow splitting and shearing at the junctions of successive elements and a stretching and folding mechanism within the elements. This makes the Kenics element an excellent radial mixing device.

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INTRODUCTION

Mixing is an operation commonly encountered in the chemical process industries. Often used mixing devices are dynamic mixers for agitated tanks and static mixers for pipeline mixing. The Kenics helical mixing element is mainly used for in-line blending of liquids under laminar flow conditions. Other types of static mixers are available for turbulent operating conditions and gas-gas mixing [1].

The Kenics in-line mixer (Figure 1) consists of a number of elements of alternating right and left hand 180 degree helices. The elements are positioned such that the leading edge of an element is perpendicular to the trailing edge of the next element. The length of the elements is one and a half tube diameters.

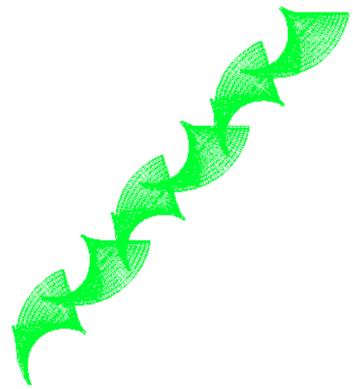

Figure 1 *The Kenics static mixer.*

Kenics in-line mixers have been used in the chemical process industries for about 30 years. Most of the experimental work concentrated on establishing design guidelines and pressure drop correlations [2-6]. The number of investigations to the flow and the mixing mechanisms is limited, probably due to experimental difficulties. The recent advancements in Computational Fluid Dynamics (CFD) have raised the question to which extent computer simulations can be used as a tool in the design and analysis of static mixers.

This work has two objectives. The first objective is to explore the possibilities Computational Fluid Dynamics offers in the analysis of mixing induced by helical elements. The second objective is to gain insight in the mixing mechanism in Kenics static mixers under laminar flow conditions. To meet these objectives the flow pattern, pressure drop and mixing characteristics of Kenics in-line mixers are analyzed by means of computer simulations with Fluent™ V4.

This article describes the numerical model, the calculated flow pattern and the mixing of two chemical species. Parts of the article were published before by Bakker and LaRoche [13] and Bakker et al. [14].

NUMERICAL MODEL

Geometry

The model consisted of a tube with a diameter of 0.02 m and a length of 0.24 m. The tube was equipped with six 180 degree elements with a length of 0.03 m each. There was an empty piece of tube at the beginning and at the end with a length of one element. The thickness of the elements was 0.04 times the tube diameter. The density of the liquid was 1000 kg/m^3. The liquid viscosity was 0.02 Pa.s. The Reynolds number was Re = 10, which is typical for the regime in which this mixer is usually used.

To evaluate the mixing in the tube, the transport of a tracer chemical species was calculated. The binary diffusion coefficient of the tracer species in the main fluid was $\mathbf{D}$ = 1.5E-9 m^2/s. This is the diffusion coefficient of saline solution in water. The center of the inlet had a tracer concentration of 100%. The outside of the inlet had a zero concentration of tracer species. The tracer fluid had the same viscosity and density as the main fluid.

Solution Method

A body fitted, structured hexahedral grid was used of approximately 100,000 grid nodes. The grid was generated with Fluent PreBFC™ V4 and exported to Fluent V4 for performing the flow and mixing computations. After initial calculations on a workstation were finished the number of internal cells in the grid was quadrupled, resulting in approximately 350,000 cells. The final calculations were performed on a Cray C90 computer.

The helical elements were modeled by blocking the flow with "wall cells". Fluid entered the tube through "inlet cells" at the inlet. A uniform velocity profile with velocities of 0.01 m/s was prescribed. The outlet of the tube was modeled by means of zero gradient boundary conditions for all flow variables.

Using the SIMPLE method and a standard line-Gauss-Seidel solver, the model turned out to be difficult to converge. Switching to a multi grid solver solved these problems and the solution process was stable. About 1000 iterations were needed to achieve convergence.

Starting the computation with a relatively accurate initial guess for the flow field had a positive effect on the speed of convergence. An initial guess was made by patching all the tangential velocities (here tangential means parallel to the grid lines in the direction of the tube) with the same velocity as prescribed in the inlet of the tube.

The flow rates on both sides of the elements should be equal due to the symmetry of the design. The model will only predict a symmetrical flow pattern when the static mixing elements have a thickness of at least two cells. When the element is only one cell thick, the flow impinging on the

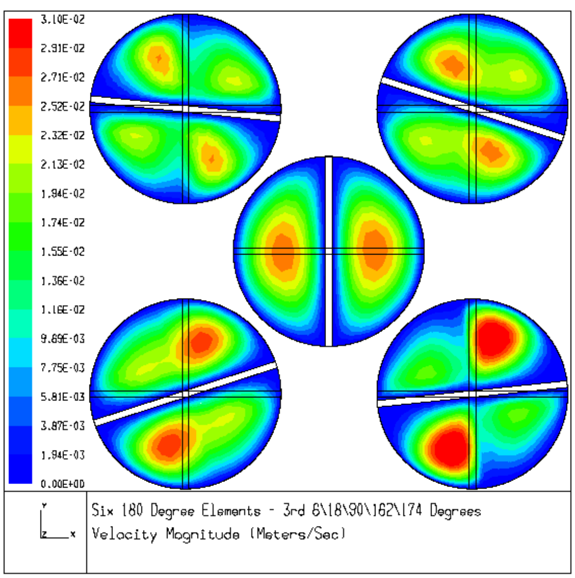

Figure 2 *Velocity magnitudes.*

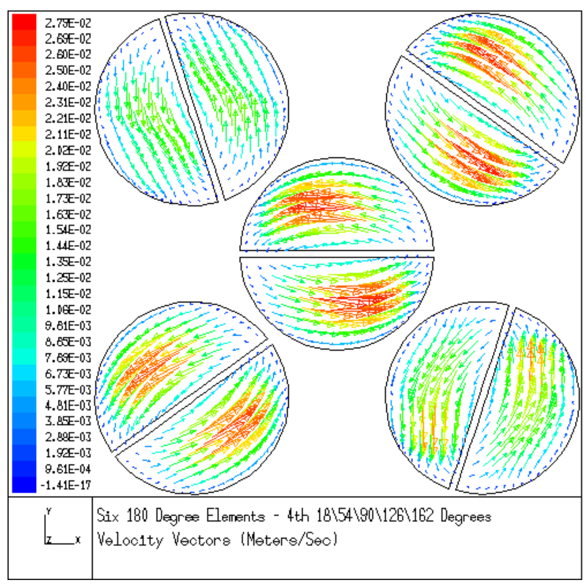

Figure 3 *Projected velocity vectors at various cross sections.*

element edge has to split between the two sides of the element in this one cell, and the split may not be even. For the correct modeling of the flow division, at least two cells are necessary. After the grid was quadrupled, the elements were 4 cells thick, giving sufficient resolution at the flow divisions.

The flow field was calculated using the power-law numerical interpolation scheme. This flow field was then used as a basis for calculating the transport of the two chemical species in the tube. To minimize the effect of false, numerical diffusion on the predicted mixing rate the QUICK numerical interpolation scheme was used in these calculations. Numerical diffusion is the smearing out of gradients due to interpolation errors or due to a too coarse grid. Numerical diffusion is less with the Quick scheme than with the Power Law interpolation scheme [7]. A disadvantage of using Quick is that the computation time is longer.

RESULTS

Flow Pattern

Figure 2 shows a raster plot of the velocity magnitude at various intersections in a tube equipped with six 180 degree elements. Red denotes high velocities and blue denotes low velocities.

At the inlet a flat velocity profile is prescribed. This profile rapidly develops into a parabolic velocity profile with higher velocities in the center then at the wall. High speed cores are formed. The high speed cores are split up at the flow divisions, resulting in four cores, arranged in a flower like pattern. Within every element those four cores merge and form two high speed cores again, one on each side of the element. Note that near the end of the elements the high speed cores are located in the corners and not in the center. The highest velocities are found more near the corners, just before the junctions. The swirl of the fluid forces more fluid to enter the next element on the downstream side than on the upstream side of the flow division.

Figures 3 and 4 show the projections of the velocity vectors at various intersections within the tubes. Figure 4 shows the velocities near a flow division.

Wilkinson and Cliff [2] state that there is a significant amount of fluid circulation within the elements. The flow simulations show that under these conditions there is no visible circulation when the fluid velocities are plotted in the Cartesian frame of reference. All the liquid velocities are directed along the helical blade, except near the junctions, where some circulation occurs. Fluid moving within an element does not seem to circulate. However, there is a relative motion between the helical blade and the fluid due to the fact that, depending on the position, the blade will twist away or towards the fluid when the fluid is moving axially through the tube. To get an impression of this relative motion a coordinate transformation to the helical reference frame has been performed, making use of the Fluent User Subroutines option. The velocities in the helical reference frame are given by:

$$u_{hr} = u - \frac{T}{l} \langle w \rangle \sqrt{x^2+y^2} \cos\left[atan\left(\frac{y}{x}\right)\right]$$

$$v_{hr} = v + \frac{T}{l} \langle w \rangle \sqrt{x^2+y^2} \sin\left[atan\left(\frac{y}{x}\right)\right] \qquad \textbf{(1)}$$

Here u and v are the velocities in the x and y directions respectively, relative to a Cartesian frame of reference; u_{hr} and v_{hr} are the corresponding velocities in the helical reference frame; $\langle w \rangle$ is the average velocity in the z direction (Cartesian reference frame, parallel with the tube); T is the twist of the element in radians and l is the length of the element.

Figure 5 shows both the absolute velocities (left, Cartesian frame of reference) and the relative velocities (right, helical reference frame). The blade is twisted in the clockwise direction. This figure illustrates that, although there is no real circulation, the position of fluid packets will shift relative to the blade while the fluid moves through the mixer.

Pressure Drop

The pressure drop across the elements was calculated with the correlation proposed in the KTEK-series [8] for laminar flow, which forms the basis for the Kenics design procedures:

$$\Delta p = (K'_{OL}A + K_{OL}) \frac{64}{Re} \frac{L}{D} \frac{1}{2} \rho \langle w \rangle^2 \qquad \textbf{(2)}$$

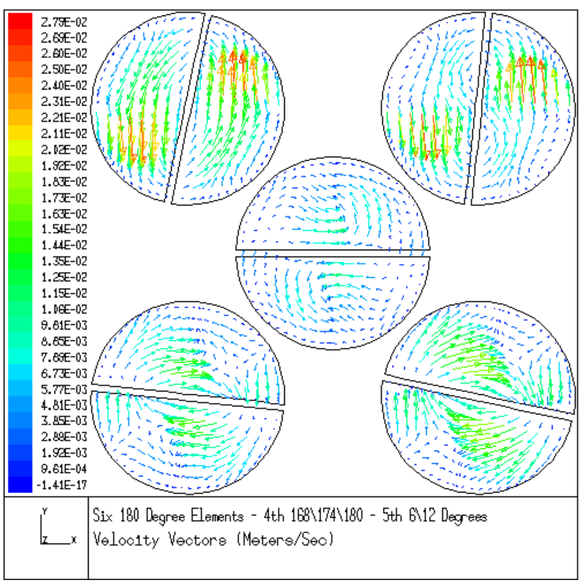

Figure 4 *Projected velocity vectors at various cross sections near a junction.*

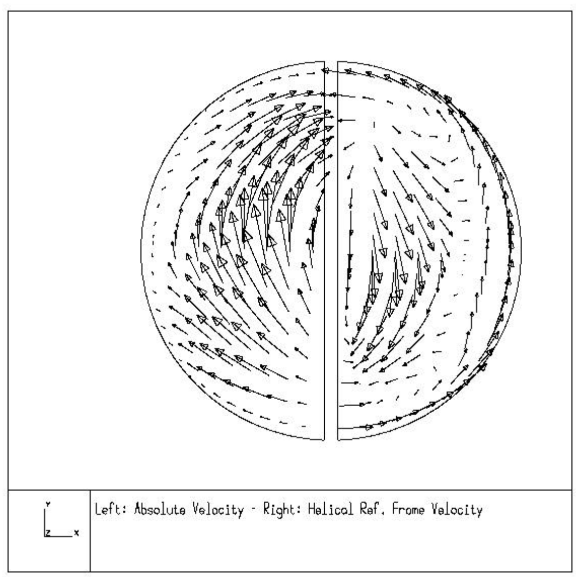

Figure 5 *Velocities in Cartesian reference frame (left) and velocities in the helical reference frame (right). Center of an element.*

The various parameters used in Equation (2) are listed in Table 1. For reasons of comparison the pressure drop was also calculated with various literature correlations. The predicted pressure drops are listed in Table 2.

The pressure drop predicted by the Kenics design procedure and predicted by Fluent are within 14%, thus giving confidence in the results. It should be kept in mind that the Kenics design procedure might be slightly conservative. There is a large variation in the pressure drop calculated by the literature correlations. However, the average for the literature correlations is within 1% of the Kenics correlation and within 13% of the Fluent prediction.

Figure 6 shows a raster plot of the pressure at the element surface. The fluid moves from the right to the left. The right side of the picture shows the pressure at the front of the element, looking at the element. The left side of the picture shows the pressure at the back of the element, looking through the element. A high pressure region is found where the high speed core coming from the previous element impinges on the blade (top right, Figure 6). A low pressure region is found where the fluid leaves the element (top left, Figure 6). The difference between the minimum and maximum pressures at the element surface was 1.8 times the average pressure drop across the element.

Mixing

The transport of a tracer species was calculated. The binary diffusion coefficient of the two species was $\mathbf{D} = 1.5.10^{-9}$ m^2/s. The center of the inlet had a concentration of tracer species of 1 (= 100%). The results are presented by means of raster plots, showing the concentration fields of the chemical species at various intersections in the tubes, see Figure 7. The color key to the tracer concentration is shown on the left. Concentrations of 0.8 (= 80%) or larger all have the same color, as do concentrations lower than 0.2 (= 20%).

Table 1 *Parameters for equation (2). A, K_{ol} and K'_{ol} are Re dependent; the values listed here are for Re = 10 only.*

L/D	Re	<w>	ρ	A	K_{ol}	K'_{ol}
9	10	0.01	1000	8.5	5.31	0.0528

Table 2 *Pressure drop across the six 180 degree elements.*

	CFD Model	Kenics [8]	Wilkinson [2]	Pahl [3]	Kemblowski [4]	Bohnet [5]	Shaw [6]
Δp (Pa)	14.4	16.6	21.7	20.1	16.3	12.9	11.7
			Average Literature: 16.5 Pa				

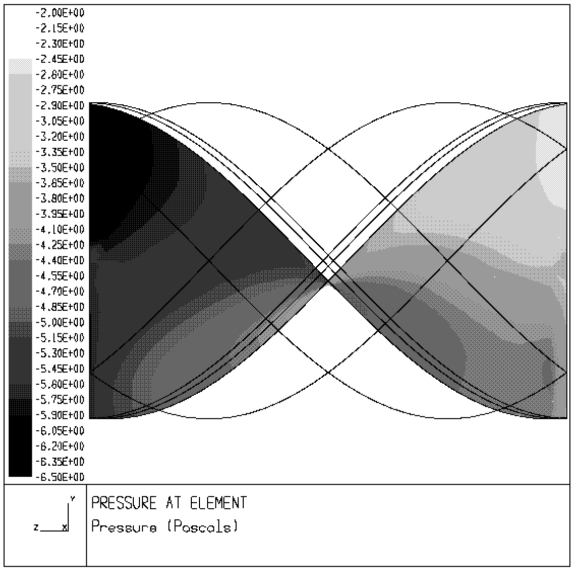

Figure 6 *Pressure at the surface of a helical element. Pressure is relative to the pressure at the inlet.*

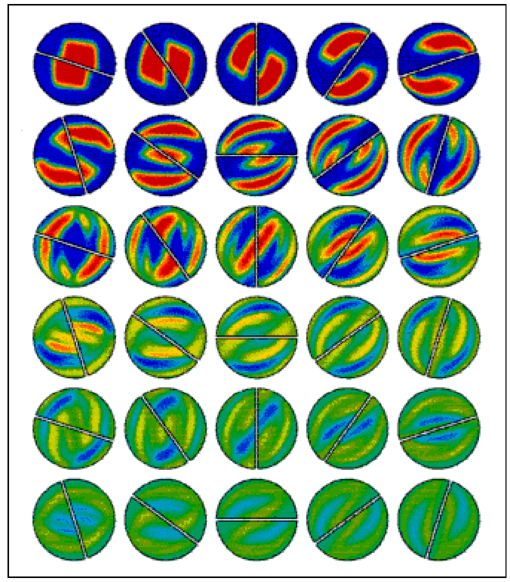

Figure 7 *Concentration profiles in the mixer. Rows one to six show the concentration in elements one to six respectively. Columns one to five show the concentration profiles at 18, 54, 90, 126 and 162 degrees respectively.*

Rows 1 to 6 in Figure 7 shows the concentration fields after 18, 54, 90, 126 and 162 degrees rotation in the first to sixth 180 degree mixing element respectively. The top row in Figure 7, showing the species concentration in the first element, shows how the high concentration core coming from the inlet is split into two high concentration islands. The two high concentration islands are stretched and move outward. The low concentration fluid, which was on the outside in the inlet of the element is split in two semi-circular filaments, which are moved towards the inside of the element.

The second element splits the two high concentration islands, second row in Figure 7, forming four high concentration islands, located relatively close to the corners near the blade. The two low concentration zones are split into four zones too, but since these were located near the centerline, parts of these low concentration zones merge. Within the element most of the low concentration fluid is to the center.

When we compare the concentration profiles at 18 degrees in the third element, first cross section in row 3, with the profile at 18 degrees in the first element (1st row) we see that we now have a low concentration fluid in the center instead of a high concentration core. The highest concentrations are now found close to the outside. The splitting and stretching process in the first two elements has resulted in a concentration field which looks like it is flipped inside-out. This process of splitting, stretching, folding and flipping inside out repeats itself every two elements, until the fluids are mixed. By the time the end of the sixth element, last row in Figure 7, is reached the species concentrations are much more uniform.

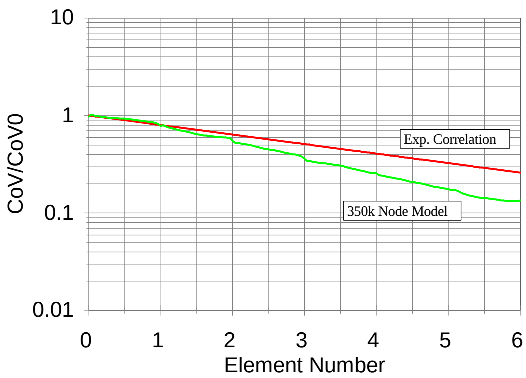

Figure 8 *Comparison of the predicted coefficient of variation with an experimental correlation.*

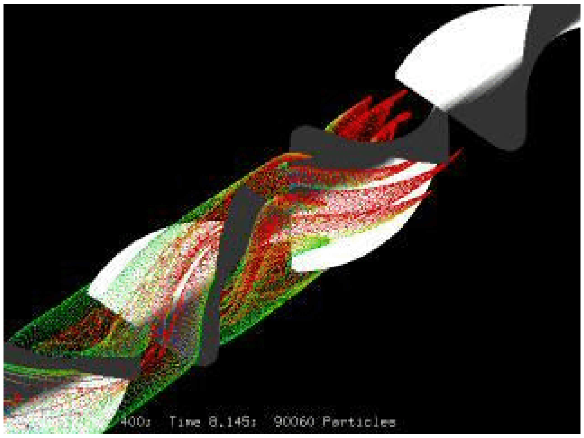

Figure 9 *Particle traces through the Kenics mixer as calculated by HyperTrace.*

Quantitative Mixing Analysis

According to Myers et al. [9], the accepted approach for determining composition uniformity in a flow field is to take simultaneous samples at various points over the conduit cross section at a fixed axial location. The most widely used measure of uniformity is the coefficient of variation, CoV, which is the ratio of the standard deviation in composition, σ, and the mean composition, x_m:

$$CoV = \frac{\sigma}{x_m} = \frac{\sqrt{\dfrac{\sum (x_i - x_m)^2}{N - 1}}}{x_m} \qquad \textbf{(3)}$$

N represents the number of data points in the sample. As a rule of thumb, most industrial blending operations can be satisfied with a coefficient of variation of five percent (COV = 0.05). However, some applications, such as the blending of colors to visual uniformity, may require coefficients of variation of one percent or less (COV $\leq$ 0.01) [10]. Experimental correlations often provide CoV relative to the CoV at the inlet, CoV_0:

$$CoV_o = \sqrt{\frac{1 - \varphi_a}{\varphi_a}} \qquad \textbf{(4)}$$

φ_a represents the ratio of the volumetric flow rate of the added material and the total volumetric flow rate:

$$\varphi_a = \frac{Q_a}{Q_t} \qquad \textbf{(5)}$$

Figure 8 shows a comparison between the coefficient of variation calculated from the 350k node model and the experimental correlation provided by Myers et al. [9]. The comparison shows that as a result of numerical diffusion, the predicted coefficient of variation decreases faster than what would be expected based on the experimental correlation. This indicates that although the flow field is calculated correctly at this grid density, a still larger grid density is needed for an accurate quantitative calculation of the degree of mixing of multiple chemical species.

However, recently higher-order particle tracking models have been developed that promise to be able to accurately predict the quantitative degree of mixing based on coarser grids than what is needed with a species transport model. Figure 9 shows particle tracks calculated using the HyperTrace™ program from SGI/Cray Research [11]. This is a promising area for future research.

DISCUSSION

The current state of the art in CFD allows for the modeling of flows and mixing of chemical species in complex geometries like the Kenics mixing element. The main drawback, however, is the long computational time and memory requirement. Calculations took about 8 hours on a Cray C-90 computer. However, even though the simulations are computer time intensive, optimizing the geometry of the mixing elements for a variety of operating conditions, fluid viscosities, equipment size etc. can be done faster this way than by conducting an extensive experimental program.

The Kenics design procedures predict a 14% higher pressure drop than the CFD simulations do, thus allowing for a safety margin.

The Fluent results indicate that under the conditions studied, in laminar flow there is no visible circulation, when the results are displayed in a Cartesian frame of reference. There is, however, a relative motion between the fluid and the mixing element due to the twist of the element, which results in effective radial mixing. Previous experimental work at Chemineer and recent work by Muzzio [12] indicates that at Reynolds numbers above approximately 400, radial circulation patterns do start to occur. Although it was beyond the original intent of this work, a calculation was performed for a fully turbulent condition, where vortex formation was observed.

The flow pattern and mixing of a chemical species in a tube equipped with Kenics elements was calculated to evaluate the mixing mechanism. Mixing occurs through a combination of flow splitting and shearing at the junctions of successive elements and a stretching and folding mechanism within the elements. The concentration field looks like it is flipped inside out after two elements: material originally at the wall is in the core and vice versa. This makes the Kenics element an excellent radial mixing device, applicable in a variety of laminar mixing applications and to improve wall to liquid heat transfer rates.

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REFERENCES

[1] Bakker A., LaRoche R. (1993)
Flow and Mixing with Kenics Static Mixers
Cray Channels, Volume 15, Number 3, page 25-28

[2] Wilkinson W.L., Cliff M.J. (1977)
An Investigation into the Performance of a Static In-Line Mixer
Proceedings Second European Conference on Mixing, 30th March - 1st April, 1977, Held at St John's College, Cambridge, England

[3] Pahl, Muschelknautz (1979)
Chem.-Ing.-Techn., 51, No. 5, p.347

[4] Kemblowski Z., Pustelnik P., Frontczak, Julianski (1976)
Reszyty Naukow Politechniki Lodzkiej, Inzynieria Chemiczne, No. 6, p. 29

[5] Bohnet M., Calvetz H. (1990)
Symp. "25 Years of Static Mixing", 16-17 October, Aachen, Germany

[6] Shaw N.F., Kale D.D. (1991)
Pressure Drop for Laminar Flow of Non-Newtonian Fluids in Static Mixers
Chem.Eng.Sci., Vol. 46, No. 8, p2159-2161

[7] Fluent V4 Users Manual (1995)
Users Guide

[8] Kenics Static Mixers KTEK-Series (1988)
Chemineer, Inc., Internal Report

[9] Myers K.J., Bakker A., Ryan D. (1997)
Avoid Agitation by Selecting Static Mixers
Chemical Engineering Progress, Vol. 93, No. 6, pp. 28-38

[10] Etchells A.W., Short D.G.R. (1988).
Pipeline Mixing - A User's View
Proc. 6th Eur. Conf. on Mixing, Pavia, Italy, IChemE, pp. 593-544

[11] http://www.cpcfd.org (1998)

[12] Muzzio F. (1997)
Private communication.

[13] Bakker A., LaRoche R.D. (1993)
Flow and Mixing with Kenics Static Mixers
Cray Channels, Vol. 15, Issue 3, pp. 25-28

[14] Bakker A., Marshall E.M. (1992)
Laminar Mixing with Kenics In-Line Mixers
Proceedings Fluent Inc.'s Users Group Meeting, 1992, pp. 126-146.

NOTATION

CoV	Coefficient of variation	(-)
CoV_0	Coefficient of variation at inlet	(-)
D	Tube diameter	(m)
D	Diffusion coefficient	(m^2s^{-1})
l	Length of a helical element	(m)
L	Length of array of helical elements	(m)
N	Number of data points in sample	(-)
Δp	Pressure drop	(Pa)
Q_a	Added flow rate	(m^3s^{-1})
Q_t	Total flow rate	(m^3s^{-1})
Re	Reynolds number	(-)
T	Twist of the blade	(radians)
u	Velocity in x-direction	(m)
v	Velocity in y-direction	(m)
w	Velocity in z-direction	(m)
<w>	Average normal velocity in a plane	($m\ s^{-1}$)
x	X-coordinate	(m)
x_i	Value of sample I.	(-)
x_m	Value of sample m	(-)
y	Y-coordinate	(m)
z	Axial coordinate	(m)
φ_a	Ratio of added flow rate to total flow rate	(-)
σ	Standard deviation	(-)